Internet Appendix

for

Common Factors, Information, and Holdings Dispersion

This Internet Appendix provides supplemental material to accompany the above titled paper.

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A Additional Notes

A.1 Notes on Positioning

Our model is presented in terms of information. Investors can be differentially informed about the asset-specific component of a stock's payoffs and/or about common components that affect the payoffs of many stocks. Given the one-period nature of the model, it is possible to also think of related frameworks.

Investors could be endowed with perceptions about certain assets. These perceptions might make investors feel some assets are safer (or more risky) than they actually are. Such a framework is discussed in Gehrig (1993). In this light, our paper differs from the earlier paper by also considering perceptions about common components.

In our opinion, and in an international context, it may make sense to think of investors having positive perceptions of their home country's assets. These perceptions would lead investors to view home-country assets as safer than they actually are. The perceptions would also lead to home bias. It is less natural, in our opinion, to think about investors having perceptions about common components. We therefore have continued the tradition of REE models and positioned the paper in terms of information dispersion and asymmetry. We acknowledge other ways one could position the paper and thank Bruno Solnik for helpful suggestions.

A.2 Correlated Uncertainties

Admati (1985) describes how one might include asset-specific information and common-component information in order to think about tradeoffs faced by investors. In this case, the tradeoffs are between stock selection and market timing. The 1985 paper also mentions the importance of trying to study such tradeoffs.

The Admati (1985) and Kodres and Pritsker (2002) models consider that assets payoffs may be correlated thanks to the correlation of residual terms (e.g., in our model, these are the terms about which nobody has information). In these cases, the residuals may have a factor structure but no investor is informed on these factors. These models, thus, do not allow studying the above-mentioned tradeoffs.

Thinking of correlated residuals, Admati (1985) clearly mentions that allowing assets to have correlated payoffs is not sufficient to allow financial economists to study the tradeoffs between stock selection and market timing. Analyzing such tradeoffs requires formalizing two distinct types of information that investors can simultaneously have. One type concerns asset-specific information and the other type concerns the common components of asset payoffs. Our model offers such an approach.

VanNieuwerburgh and Veldkamp (2010) consider that investors may have information about the asset-specific component of payoffs or, alternatively, about the common component of payoffs. They study, in detail, cases in which asset payoffs are not correlated and investors are informed about asset-specific components of stocks payoffs. They also mention that when the payoffs are correlated the covariance matrix could be factored such that investors have information about the common components that generate returns. However, there is no mention in the paper about investors being informed simultaneously about both asset-specific and common components. Nor, is there a mention about investors making a tradeoff between these two components. Allowing such a tradeoff is precisely what our paper does.

B Equations and Proofs

Our Manuscript and Admati (1985)

We compare and contrast the model in our manuscript with the model in the Admati (1985) paper. While some equations may initially appear similar across the two, the model we propose is based on different assumptions and hypotheses. Recognizing the differences in assumptions and hypotheses is key to understanding the similarities and differences in the proofs.[1]

B.1 Similarities

To start, let's look at a clear similarity. Our model is a static, rational expectations equilibrium model as so is the model in 1985 paper. This form of modeling implies that the prices of the risky assets play two roles:

a) They equate supplies and demands for each of the risky assets.

b) They transmit private information from the informed investors to the uninformed investors. A given investor's expected utility of final wealth is conditioned by his or her private information as well as by the equilibrium prices.

We agree that Equations (11), (10), and (12) in the 1985 paper are similar to our respective Equations (5), (7), and (6)—from our main paper. Finally, the 1985 proof of Lemma 3.2 seen in Equation (17) to Equation (20), is similar to the steps shown in our proof in Appendix D. While the aforementioned equations are similar, there are significant differences between the 1985 paper and our manuscript. We turn to exploring those differences.

B.2 Differences

Let's look at differences between our model's equations (and the associated proofs) and those in the 1985 paper. The 1985 paper assumes that each investor receives his or her own vector of signals about the payoffs of the risky assets. In the 1985 paper, the signal is modeled as being equal to the actual future payoff plus an error term. In contrast, we assume the existence, for each asset and for each common component, of two groups of investors. There are the informed investors who observe the same signal about the asset or common

[1] Before getting into a detailed analysis of differences, please note that the notations (i.e., the name of the variables and matrices) are not the same across the two models.

component. We too model a signal as equal to a future value plus an error term. Our model also includes uninformed investors who do not receive a signal, but instead only observe the prices of the risky assets.

A second clear difference is that the 1985 paper assumes the existence of a large market in order to find closed form solutions for the equilibrium prices. The assumption of a large market implies that the error terms in each investor's signal vector vanish by the law of large numbers.[2] The equilibrium price vector is thus a linear function of the future payoff and a random supply shock. In contrast, we do not make such an assumption and have aggregate residual uncertainty about each asset's payoff. In our framework and economically, the noise in investors' signals do not cancel. The aggregate amount of noise (residual uncertainty) affects asset prices for risk-averse investors.

Why do our signals more closely resemble those in a Grossman-Stiglitz-type framework than signals in the Admati (1985) model? There are at least two reasons for our modeling choice. First, we do not believe that an investor who can aggregate all signals (about a given risky asset) can eliminate uncertainty and be left holding a riskless asset. Instead, we prefer the notion that some degree of uncertainty remains no matter how efficiently an investor may aggregate signals. Second, while the modeling choices in the 1985 paper help to find closed-form solutions, the choices appear to block one's ability to obtain closed-form solutions when there are common components.[3] As far as we know, our modeling choice is needed in order to find a closed-form solution for a model with common components.

Next, let's look at two equations that appear similar, but actually represent different underlying variables. Equation (6) in the 1985 paper and our Equation (4) from our main paper look similar at first glance. However, the equilibrium price in the 1985 paper is a linear function of future payoffs and supplies. The equilibrium price in our model is a linear function of the asset-specific components vector, the global components vector, and the supply vector. To summarize, the earlier paper includes future payoffs; our model includes components of future payoffs.

We agree that Equations (13), (14), and (15) in the 1985 paper appear similar to our Equation (8) from our main paper. There are, again, differences between the sets of equations. The main difference is the $\mathbf{M}_n$ matrix as seen in our Equation (8) from our main paper. This

[2] As stated on page 655 of the 1985 paper, there is no residual uncertainty in the large market. For an investor who possesses all the information signals, the risky assets become riskless.

[3] Please see p.653 of Admati (1985) where it is noted that a related model, but empirically distinguishable from the 1985 model, supposes payoffs are generated by a factor model. The author note that such a setting would be "very natural for the timing-selectivity distinction." However she also points out the difficulty to find a closed form solution with such a factor structure. It is mentioned in the 1985 paper (see p. 653) "Unfortunately, this model does not seem to admit a closed form solution, since it involves a system of cubic equations".

matrix represents the asset specific and the common components about which investor n is informed. Such a matrix does not exist in the 1985 model since the earlier model contains an assumption that all agents receive a signal about each and every asset. The $\mathbf{M}_n$ matrix represents the main source of differences between our model and the 1985 model.

In order to find the closed form solutions for A0, A1, and A2, the 1985 paper relies on Equation (21). In order to find the closed form solutions for A0, A1, and A2, as presented in Theorem 1, we need to first prove Lemma 1 in Appendix E. Comparing the proofs, we believe ours is new and significantly different. Our proof, as presented in Appendix E, was difficult and does not have a corresponding proof in the 1985 paper. If we study Equation (17) in the 1985 paper and Equation (4) in our Appendix D, the following differences cause mathematical complexity in our proofs:

- We consider groups of differently informed investors (all investors do not have the same variance covariance matrix of the future payoffs). In a multi-asset setting with non-commutative matrices, this makes the equations complicated to simplify.

- We have aggregate residual uncertainty about each asset and each common component. Mathematically, this implies that a variance covariance matrix of residuals (sigma epsilon) will be present in solutions.

Following a similar line of reasoning as laid out directly above, our Lemma 1 proof is complicated by the rich information structure in our model.

Finally, Equations (7) to (9) in the 1985 paper give the closed form solutions for A0, A1, and A2. The corresponding equations in our main paper are Equations (11) to (13), but they are not the same as in the 1985 model. The differences are important:

- In our model, the information is widespread among groups of investors. The matrix D plays an important role in our model. An equivalent concept does not exist in the 1985 paper.

- In our model, the market as a whole does not know the future payoff of the risky assets (our equations are functions of the expected information and not of the expected payoff).

- In our model, the residual uncertainty on the market (the matrix sigma epsilon) exists. As discussed earlier, we do not model a large market in which aggregate uncertainty goes to zero.

B.3 Similarities and Differences

This Internet Appendix has arranged arguments first into a "similarities" discussion and later into a "differences" discussion. Deeper thought will reveal that equations in the 1985 paper and our manuscript cannot be so easily classified. For example, Equation (10) in the 1985 paper does look similar to Equation (7) in our main paper. However, the equations are quite different since Equation (7) from our main paper includes an "$\mathbf{M}_n$" term that does not appear in the earlier paper.

We acknowledge that comparing and contrasting the two models is difficult. This Internet Appendix attempts to highlight key differences.

B.4 Final Thoughts

To conclude, we present another way to broadly contrast the 1985 paper with our manuscript. We think about the motivations behind the different models. The earlier paper mentions, on page 653, that a worthwhile and interesting model "would permit different agents to obtain information about different portfolios." Our manuscript essentially allows agents to obtain information about different portfolios, though doing so proves difficult and is what we believe highlights our contribution.

Thinking about our framework, we consider situations in which different agents receive information about different common components. In an extreme world with little-to-no asset-specific information, our model considers situations in which different groups of investors receive information about different portfolios (where a portfolio consists of all stocks affected by a given common component). Of course, our model is more complicated than the simply modeling of the extreme world mentioned above. We consider asset-specific information in addition to information about common components.

C Holdings of Investor Group n

The information set of investor n (formally, investor i in group n) consists of the realization of private signals $\mathbf{M}_n\tilde{\eta}$ and of equilibrium prices $\tilde{P}^0$. The equilibrium price vector $\tilde{P}^0$ is a linear function of the information $\tilde{\eta}$ and the supply $\tilde{z}$ with $\tilde{P}^0 = A_0 + \mathbf{A}_1\tilde{\eta} - \mathbf{A}_2\tilde{z}$. Since, $\tilde{w}_n^1 = w_n^0 R + X_n'(\tilde{P}^1 - R\tilde{P}^0)$ and $\tilde{P}^1$ is a linear function of $\tilde{\eta}$ and $\tilde{\varepsilon}$, it follows that $\tilde{w}_n^1$ joins the multivariate normal distribution of $(\tilde{\eta}, \tilde{\varepsilon}, \tilde{z})$. Consequently, $\tilde{w}_n^1$ is a normal random variable conditional on $\mathbf{M}_n\tilde{\eta}$ and $\tilde{P}^0$. Properties of normal distributions imply that investor n's expected utility can be written as:

$$
\begin{aligned}
E\left[U(\tilde{w}_n^1)|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] &= U\left\{E\left[\tilde{w}_n^1|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] - \frac{a}{2}Var\left[\tilde{w}_n^1|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right]\right\} \\
&= U\Big\{E\left[w_n^0 R + X_n'(\tilde{P}^1 - R\tilde{P}^0)|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] - \frac{a}{2}Var\left[w_n^0 R + X_n'(\tilde{P}^1 - R\tilde{P}^0)|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right]\Big\}
\end{aligned}
$$

Since the utility function is exponential, maximizing this expected utility is identical to maximizing:

$$
\begin{aligned}
&\max_{X_n}\left\{E\left[w_n^0 R + X_n'(\tilde{P}^1 - R\tilde{P}^0)|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] - \frac{a}{2}Var\left[w_n^0 R + X_n'(\tilde{P}^1 - R\tilde{P}^0)|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right]\right\} \\
&\quad = \max_{X_n}\left\{X_n' E\left[\left(\tilde{P}^1 - R\tilde{P}^0\right)|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] - \frac{a}{2}X_n' Var\left[(\tilde{P}^1 - R\tilde{P}^0)|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] X_n\right\}
\end{aligned}
$$

The equation to be solved is:

$$
0 = E\left[(\tilde{P}^1 - R\tilde{P}^0)|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] - aVar\left[(\tilde{P}^1 - R\tilde{P}^0)|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] X_n \tag{1}
$$

This implies that investor n's demand vector is:

$$
\tilde{X}_n = a^{-1}Var^{-1}\left[\tilde{P}^1|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] \times \left(E\ \left[\tilde{P}^1|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] - R\tilde{P}^0\right) \tag{2}
$$

D System to be Solved

From Equation (4) from the main paper, Equation (5) from the main paper, and Equation (6) from the main paper, we have:

$$0 = \sum_{n=1}^{N} \lambda_n \mathbf{V}_n^{-1} \Big(B_{0n} + \mathbf{B}_{1n} \mathbf{M}_n \tilde{\eta} + \big(\mathbf{B}_{2n} - R \mathbf{I}_J \big) \big(A_0 + \mathbf{A}_1 \tilde{\eta} - \mathbf{A}_2 \tilde{z} \big) \Big) - a \tilde{z}$$

By canceling the $\tilde{z}$, $\tilde{\eta}$, and constant terms, it is straightforward to show that:

$$\begin{aligned} a \mathbf{A}_2^{-1} A_0 &= \sum_{n=1}^{N} \lambda_n \mathbf{V}_n^{-1} B_{0n} \\ a \mathbf{A}_2^{-1} \mathbf{A}_1 &= \sum_{n=1}^{N} \lambda_n \mathbf{V}_n^{-1} \mathbf{B}_{1n} \mathbf{M}_n \\ a \mathbf{A}_2^{-1} &= \sum_{n=1}^{N} \lambda_n \mathbf{V}_n^{-1} \left(R \mathbf{I}_J - \mathbf{B_{2n}} \right) \end{aligned} \tag{3}$$

The vector $\begin{pmatrix} \tilde{P}^{1\prime} & \mathbf{M}_n \tilde{\eta}^{\prime} & \tilde{P}^{0\prime} \end{pmatrix}^{\prime}$ is normally distributed and its var-cov matrix is:

$$Var\left[\begin{pmatrix} \tilde{P}^{1\prime} & \mathbf{M}_n \tilde{\eta}^{\prime} & \tilde{P}^{0\prime} \end{pmatrix}^{\prime} \right] = \begin{pmatrix} \mathbf{CQC}^{\prime} + \mathbf{\Sigma}_{\varepsilon} & \mathbf{CQM}_n^{\prime} & \mathbf{CQA}_1^{\prime} \\ \mathbf{M}_n \mathbf{QC}^{\prime} & \mathbf{M}_n \mathbf{QM}_n^{\prime} & \mathbf{M}_n \mathbf{QA}_1^{\prime} \\ \mathbf{A}_1 \mathbf{QC}^{\prime} & \mathbf{A}_1 \mathbf{QM}_n^{\prime} & \mathbf{A}_1 \mathbf{QA}_1^{\prime} + \mathbf{A}_2 \mathbf{\Sigma}_z \mathbf{A}_2^{\prime} \end{pmatrix} \tag{4}$$

The conditional expectation is:

$$\begin{aligned} E_n \left[\tilde{P}^1 \right] &= E \left[\tilde{P}^1 | \mathbf{M}_n \tilde{\eta}, \tilde{P}^0 \right] \\ &= E[\tilde{P}^1] + Cov \left[\tilde{P}^1 ; \begin{pmatrix} \mathbf{M}_n \tilde{\eta} \\ \tilde{P}^0 \end{pmatrix} \right] \times Var^{-1} \begin{bmatrix} \mathbf{M}_n \tilde{\eta} \\ \tilde{P}^0 \end{bmatrix} \times \left(\begin{pmatrix} \mathbf{M}_n \tilde{\eta} \\ \tilde{P}^0 \end{pmatrix} - E \begin{bmatrix} \mathbf{M}_n \tilde{\eta} \\ \tilde{P}^0 \end{bmatrix} \right) \end{aligned}$$

Normal distributions give $E \left[\tilde{P}^1 | \mathbf{M}_n \tilde{\eta}, \tilde{P}^0 \right] = B_{0n} + \mathbf{B}_{1n} \mathbf{M}_n \tilde{\eta} + \mathbf{B}_{2n} \tilde{P}^0$. Hence,

$$\begin{pmatrix} \mathbf{B}_{1n} & \mathbf{B}_{2n} \end{pmatrix} = Cov \left[\tilde{P}^1 ; \begin{pmatrix} \mathbf{M}_n \tilde{\eta} \\ \tilde{P}^0 \end{pmatrix} \right] \times Var^{-1} \begin{bmatrix} \mathbf{M}_n \tilde{\eta} \\ \tilde{P}^0 \end{bmatrix} \tag{5}$$

$$\begin{pmatrix} \mathbf{CQM}_n^{\prime} & \mathbf{CQA}_1^{\prime} \end{pmatrix} = \begin{pmatrix} \mathbf{B}_{1n} & \mathbf{B}_{2n} \end{pmatrix} \begin{pmatrix} \mathbf{M}_n \mathbf{QM}_n^{\prime} & \mathbf{M}_n \mathbf{QA}_1^{\prime} \\ \mathbf{A}_1 \mathbf{QM}_n^{\prime} & \mathbf{A}_1 \mathbf{QA}_1^{\prime} + \mathbf{A}_2 \mathbf{\Sigma}_z \mathbf{A}_2^{\prime} \end{pmatrix} \tag{6}$$

The variance of returns conditional on n's information is:

$$\begin{aligned}\mathbf{V}_n &= Var\left[\tilde{P}^1|\mathbf{M}_n\tilde{\eta},\tilde{P}^0\right] \\ &= Var\left[\tilde{P}^1\right] - Cov\left[\tilde{P}^1;\begin{pmatrix}\mathbf{M}_n\tilde{\eta}\\ \tilde{P}^0\end{pmatrix}\right]\times Var^{-1}\begin{bmatrix}\mathbf{M}_n\tilde{\eta}\\ \tilde{P}^0\end{bmatrix}\times Cov\left[\begin{pmatrix}\mathbf{M}_n\tilde{\eta}\\ \tilde{P}^0\end{pmatrix};\tilde{P}^1\right]\end{aligned} \tag{7}$$

We use Equation (5) and Equation (7) to get:

$$\mathbf{V}_n = Var\left[\tilde{P}^1\right] - \begin{pmatrix}\mathbf{B}_{1n} & \mathbf{B}_{2n}\end{pmatrix} Cov\left[\begin{pmatrix}\mathbf{M}_n\tilde{\eta}\\ \tilde{P}^0\end{pmatrix};\tilde{P}^1\right]$$

Because $Cov\left[\begin{pmatrix}\mathbf{M}_n\tilde{\eta}\\ \tilde{P}^0\end{pmatrix};\tilde{P}^1\right] = \begin{pmatrix}\mathbf{M}_n\mathbf{QC}'\\ \mathbf{A}_1\mathbf{QC}'\end{pmatrix}$ we get:

$$\mathbf{V}_n = \mathbf{CQC}' + \mathbf{\Sigma}_\varepsilon - \mathbf{B}_{1n}\mathbf{M}_n\mathbf{QC}' - \mathbf{B}_{2n}\mathbf{A}_1\mathbf{QC}' \tag{8}$$

E Proof of Lemma 1

In order to determine a closed-form solution for $\mathbf{U}$, we solve the second equation from the system shown in Equation (8) from the main paper:

$$a\mathbf{A}_2^{-1}\mathbf{A}_1 = a\mathbf{U} = \sum_{n=1}^{N} \lambda_n \mathbf{V}_n^{-1}\mathbf{B}_{1n}\mathbf{M}_n \tag{9}$$

The following properties apply to matrices $\mathbf{D}_n$ and $\mathbf{M}_n$. Below we use n_a and n_b to denote two different groups of investors such that $n = \{1, 2, 3, \ldots, n_a, \ldots, n_b, \ldots, N\}$:

P1: $\sum_{n=1}^{N} \mathbf{D}_n = \mathbf{I}_{J+K}$

P2: $\forall\ n_a \neq n_b$: $\mathbf{D}_{n_a}\mathbf{D}_{n_b} = \mathbf{0}_{J+K}$ where $\mathbf{0}_{J+K}$ is the null matrix of order $J+K$

P3: $\mathbf{M}_{n_a}\mathbf{M}'_{n_b} = \mathbf{0}_{Jn_a+Kn_a, Jn_b+Kn_b}$ where $\mathbf{0}_{Jn_a+Kn_a, Jn_b+Kn_b}$ is the null matrix

P4: $\mathbf{D}_n\mathbf{D}_n = \mathbf{D}_n$ and $\mathbf{M}_n\mathbf{M}'_n = \mathbf{I}_{Jn+Kn}$

P5: $\forall\ \mathbf{G_1}, \mathbf{G_2}$: $g(\mathbf{G_1})g(\mathbf{G_2}) = \sum_{n=1}^{N} \mathbf{D}_n\mathbf{G_1}\mathbf{D}_n\mathbf{G_2}\mathbf{D}_n$

P6: $\forall\ \mathbf{G}$: $g(\mathbf{GD}) = g(\mathbf{G})\mathbf{D} = \mathbf{D}g(\mathbf{G})$

There are three matrices key to obtaining a closed-form solution for $\mathbf{U}$:

$$\begin{aligned} \mathbf{M} &= \mathbf{UQU}' + \mathbf{\Sigma}_z \\ \mathbf{\Psi} &= Var\left[\tilde{\eta}|\tilde{P}^0\right] = \mathbf{Q} - \mathbf{QU}'\mathbf{M}^{-1}\mathbf{UQ} \\ \mathbf{\Psi}_n &= \mathbf{M}_n\mathbf{\Psi}\mathbf{M}'_n \end{aligned}$$

We first solve Equation (6) for $\mathbf{B}_{1n}$ and $\mathbf{B}_{2n}$. Note, we do not assume $\mathbf{A}_1$ is invertable to get the following results (in fact, $\mathbf{A}_1$ is not square). The two equations to be solved are:

$$\mathbf{B}_{1n}\left(\mathbf{M}_n\mathbf{QM}'_n\right) + \mathbf{B}_{2n}\left(\mathbf{A}_1\mathbf{QM}'_n\right) = \mathbf{CQM}'_n \tag{10}$$

$$\mathbf{B}_{1n}\left(\mathbf{M}_n\mathbf{QA}'_1\right) + \mathbf{B}_{2n}\left(\mathbf{A}_1\mathbf{QA}'_1 + \mathbf{A}_2\mathbf{\Sigma}_z\mathbf{A}'_2\right) = \mathbf{CQA}'_1 \tag{11}$$

Using $\mathbf{M} = \mathbf{UQU}' + \mathbf{\Sigma}_z$, we obtain $\mathbf{A}_1\mathbf{QA}'_1 + \mathbf{A}_2\mathbf{\Sigma}_z\mathbf{A}'_2 = \mathbf{A}_2\mathbf{MA}'_2$. Starting with Equation (11), we get:

$$\begin{aligned} \mathbf{B}_{1n}\left(\mathbf{M}_n\mathbf{QA}'_1\right) + \mathbf{B}_{2n}\left(\mathbf{A}_1\mathbf{QA}'_1 + \mathbf{A}_2\mathbf{\Sigma}_z\mathbf{A}'_2\right) &= \mathbf{CQA}'_1 \\ \mathbf{B}_{1n}\left(\mathbf{M}_n\mathbf{QA}'_1\right) + \mathbf{B}_{2n}\left(\mathbf{A}_2\mathbf{MA}'_2\right) &= \mathbf{CQA}'_1 \\ \left(\mathbf{CQA}'_1 - \mathbf{B}_{1n}\mathbf{M}_n\mathbf{QA}'_1\right)\left(\mathbf{A}_2\mathbf{MA}'_2\right)^{-1} &= \mathbf{B}_{2n} \\ \left(\mathbf{C} - \mathbf{B}_{1n}\mathbf{M}_n\right)\mathbf{Q}\left(\mathbf{A}_2^{-1}\mathbf{A}_1\right)'\mathbf{M}^{-1}\mathbf{A}_2^{-1} &= \mathbf{B}_{2n} \\ \left(\mathbf{C} - \mathbf{B}_{1n}\mathbf{M}_n\right)\mathbf{QU}'\mathbf{M}^{-1}\mathbf{A}_2^{-1} &= \mathbf{B}_{2n} \end{aligned}$$

In a second step, we solve Equation (10):

$$\begin{aligned}
\mathbf{B}_{1n}(\mathbf{M}_n\mathbf{Q}\mathbf{M}'_n) + (\mathbf{C} - \mathbf{B}_{1n}\mathbf{M}_n)\mathbf{Q}\mathbf{U}'\mathbf{M}^{-1}\mathbf{A}_2^{-1}(\mathbf{A}_1\mathbf{Q}\mathbf{M}'_n) &= \mathbf{C}\mathbf{Q}\mathbf{M}'_n \\
\mathbf{B}_{1n}\mathbf{M}_n(\mathbf{Q}\mathbf{M}'_n - \mathbf{Q}\mathbf{U}'\mathbf{M}^{-1}\mathbf{U}\mathbf{Q}\mathbf{M}'_n) &= (\mathbf{C}\mathbf{Q} - \mathbf{C}\mathbf{Q}\mathbf{U}'\mathbf{M}^{-1}\mathbf{U}\mathbf{Q})\mathbf{M}'_n \\
\mathbf{B}_{1n}\mathbf{M}_n(\mathbf{Q} - \mathbf{Q}\mathbf{U}'\mathbf{M}^{-1}\mathbf{U}\mathbf{Q})\mathbf{M}'_n &= \mathbf{C}(\mathbf{Q} - \mathbf{Q}\mathbf{U}'\mathbf{M}^{-1}\mathbf{U}\mathbf{Q})\mathbf{M}'_n \\
\mathbf{B}_{1n}\mathbf{M}_n\boldsymbol{\Psi}\mathbf{M}'_n &= \mathbf{C}\boldsymbol{\Psi}\mathbf{M}'_n \\
\mathbf{B}_{1n}\boldsymbol{\Psi}_n &= \mathbf{C}\boldsymbol{\Psi}\mathbf{M}'_n \\
\mathbf{B}_{1n} &= \mathbf{C}\boldsymbol{\Psi}\mathbf{M}'_n\boldsymbol{\Psi}_n^{-1}
\end{aligned}$$

We have thus demonstrated that:

$$\begin{aligned}
\mathbf{B}_{1n} &= \mathbf{C}\Psi\mathbf{M}'_n\boldsymbol{\Psi}_n^{-1} \\
\mathbf{B}_{2n} &= \big(\mathbf{C} - \mathbf{B}_{1n}\mathbf{M}_n\big)\mathbf{Q}\mathbf{U}'\mathbf{M}^{-1}\mathbf{A}_2^{-1}
\end{aligned} \tag{12}$$

By substituting $\mathbf{B}_{1n}$ and $\mathbf{B}_{2n}$ into Equation (8) we obtain the variance-covariance matrix $\mathbf{V}_n$ as a function of $\boldsymbol{\Psi}$

$$\begin{aligned}
\mathbf{V}_n &= \mathbf{C}\mathbf{Q}\mathbf{C}' + \boldsymbol{\Sigma}_\varepsilon - \mathbf{B}_{1n}\mathbf{M}_n\mathbf{Q}\mathbf{C}' - \mathbf{B}_{2n}\mathbf{A}_1\mathbf{Q}\mathbf{C}' \\
\mathbf{V}_n &= \mathbf{C}\mathbf{Q}\mathbf{C}' + \boldsymbol{\Sigma}_\varepsilon - \mathbf{C}\boldsymbol{\Psi}\mathbf{M}'_n\boldsymbol{\Psi}_n^{-1}\mathbf{M}_n\mathbf{Q}\mathbf{C}' - (\mathbf{C} - \mathbf{C}\boldsymbol{\Psi}\mathbf{M}'_n\boldsymbol{\Psi}_n^{-1}\mathbf{M}_n)\mathbf{Q}\mathbf{U}'\mathbf{M}^{-1}\mathbf{A}_2^{-1}\mathbf{A}_1\mathbf{Q}\mathbf{C}' \\
\mathbf{V}_n &= \boldsymbol{\Sigma}_\varepsilon + \mathbf{C}(\mathbf{Q} - \mathbf{Q}\mathbf{U}'\mathbf{M}^{-1}\mathbf{U}\mathbf{Q})\mathbf{C}' - \mathbf{C}\boldsymbol{\Psi}\mathbf{M}'_n\boldsymbol{\Psi}_n^{-1}\mathbf{M}_n\mathbf{Q}\mathbf{C}' + \mathbf{C}\boldsymbol{\Psi}\mathbf{M}'_n\boldsymbol{\Psi}_n^{-1}\mathbf{M}_n\mathbf{Q}\mathbf{U}'\mathbf{M}^{-1}\mathbf{U}\mathbf{Q}\mathbf{C}' \\
\mathbf{V}_n &= \boldsymbol{\Sigma}_\varepsilon + \mathbf{C}\boldsymbol{\Psi}\mathbf{C}' - \mathbf{C}\boldsymbol{\Psi}\mathbf{M}'_n\boldsymbol{\Psi}_n^{-1}\mathbf{M}_n(\mathbf{Q} - \mathbf{Q}\mathbf{U}'\mathbf{M}^{-1}\mathbf{U}\mathbf{Q})\mathbf{C}' \\
\mathbf{V}_n &= \boldsymbol{\Sigma}_\varepsilon + \mathbf{C}\boldsymbol{\Psi}\mathbf{C}' - \mathbf{C}\boldsymbol{\Psi}\mathbf{M}'_n\boldsymbol{\Psi}_n^{-1}\mathbf{M}_n\boldsymbol{\Psi}\mathbf{C}' \qquad (13)
\end{aligned}$$

We use Equation (9) to determine $\mathbf{U}$ by right-multiplying by $\mathbf{M}'_n$. Note that $\mathbf{M}'_n$ concerns investor group n. Property P3, from the start of this appendix, shows we need only carry out the multiplication with terms from the same investor group.

Thus, we obtain $\lambda_n\mathbf{V}_n^{-1}\mathbf{B}_{1n} = a\mathbf{U}\mathbf{M}'_n$. Also from P3, $\mathbf{M}_{n_a}\mathbf{M}'_{n_b} = \mathbf{0}$. We then multiply this last expression by $\mathbf{V}_n$ on the left and we replace $\mathbf{B}_{1n}$ with its value from (12):

$$\lambda_n\mathbf{C}\boldsymbol{\Psi}\mathbf{M}'_n\boldsymbol{\Psi}_n^{-1} = a\mathbf{V}_n\mathbf{U}\mathbf{M}'_n \tag{14}$$

If we multiply (14) by $\mathbf{M}_n$ on the right and if we sum for $n = 1, \dots N$, we obtain Equation (9). We conclude that Equation (9) is equivalent to Equation (14) for all $n = 1, \dots, N$. If we multiply Equation (14) by $\boldsymbol{\Psi}_n$ and $\mathbf{M}_n$ on the right and if we replace $\mathbf{V}_n$ with its value in Equation (13) we then obtain:

$$\lambda_n\mathbf{C}\boldsymbol{\Psi}\mathbf{D}_n = a\left(\boldsymbol{\Sigma}_\varepsilon + \mathbf{C}\boldsymbol{\Psi}\mathbf{C}' - \mathbf{C}\boldsymbol{\Psi}\mathbf{M}'_n\boldsymbol{\Psi}_n^{-1}\mathbf{M}_n\boldsymbol{\Psi}\mathbf{C}'\right)\mathbf{U}\mathbf{M}'_n\boldsymbol{\Psi}_n\mathbf{M}_n$$

If we now sum for $n = 1, \ldots, N$ we obtain:

$$\sum_{n=1}^{N} \lambda_n \mathbf{C}\boldsymbol{\Psi}\mathbf{D}_n = a\left(\boldsymbol{\Sigma}_\varepsilon \sum_{n=1}^{N} \mathbf{U}\mathbf{M}_n'\boldsymbol{\Psi}_n\mathbf{M}_n + \mathbf{C}\boldsymbol{\Psi}\mathbf{C}' \sum_{n=1}^{N} \mathbf{U}\mathbf{M}_n'\boldsymbol{\Psi}_n\mathbf{M}_n - \sum_{n=1}^{N} \mathbf{C}\boldsymbol{\Psi}\mathbf{M}_n'\boldsymbol{\Psi}_n^{-1}\mathbf{M}_n\boldsymbol{\Psi}\mathbf{C}'\mathbf{U}\mathbf{M}_n'\boldsymbol{\Psi}_n\mathbf{M}_n\right)$$

which is equivalent to:

$$\mathbf{C}\boldsymbol{\Psi}\mathbf{D} = a\left(\boldsymbol{\Sigma}_\varepsilon \mathbf{U} \sum_{n=1}^{N} \mathbf{D}_n\boldsymbol{\Psi}\mathbf{D}_n + \mathbf{C}\boldsymbol{\Psi}\mathbf{C}'\mathbf{U} \sum_{n=1}^{N} \mathbf{D}_n\boldsymbol{\Psi}\mathbf{D}_n - \mathbf{C}\boldsymbol{\Psi} \sum_{n=1}^{N} \mathbf{M}_n'\boldsymbol{\Psi}_n^{-1}\mathbf{M}_n\boldsymbol{\Psi}\mathbf{C}'\mathbf{U}\mathbf{M}_n'\boldsymbol{\Psi}_n\mathbf{M}_n\right)$$

By introducing the function $g(\cdot)$, we obtain the expression below. The reader can easily check that (15) is equivalent to (9).

$$\mathbf{C}\boldsymbol{\Psi}\mathbf{D} = a\boldsymbol{\Sigma}_\varepsilon \mathbf{U} g(\boldsymbol{\Psi}) + a\mathbf{C}\boldsymbol{\Psi}\mathbf{C}'\mathbf{U} g(\boldsymbol{\Psi}) - a\mathbf{C}\boldsymbol{\Psi}\left(\sum_{n=1}^{N} \mathbf{M}_n'\boldsymbol{\Psi}_n^{-1}\mathbf{M}_n\boldsymbol{\Psi}\mathbf{C}'\mathbf{U}\mathbf{M}_n'\boldsymbol{\Psi}_n\mathbf{M}_n\right) \tag{15}$$

To prove Lemma 1, we start by assuming $\mathbf{U} = a^{-1}\boldsymbol{\Sigma}_\varepsilon^{-1}\mathbf{C}\mathbf{D}$. We then substitute this expression for $\mathbf{U}$ into (15) and check that the following equality holds:

$$\mathbf{C}\boldsymbol{\Psi}\mathbf{D} = \mathbf{C}\mathbf{D} g(\boldsymbol{\Psi}) + \mathbf{C}\boldsymbol{\Psi}\mathbf{C}'\boldsymbol{\Sigma}_\varepsilon^{-1}\mathbf{C}\mathbf{D} g(\boldsymbol{\Psi}) - \mathbf{C}\boldsymbol{\Psi}\left(\sum_{n=1}^{N} \mathbf{M}_n'\boldsymbol{\Psi}_n^{-1}\mathbf{M}_n\boldsymbol{\Psi}\mathbf{C}'\boldsymbol{\Sigma}_\epsilon^{-1}\mathbf{C}\mathbf{D}\mathbf{M}_n'\boldsymbol{\Psi}_n\mathbf{M}_n\right) \tag{16}$$

Under the assumption that $\boldsymbol{\Psi}^{-1} + \mathbf{C}'\boldsymbol{\Sigma}_\varepsilon^{-1}\mathbf{C}$ is a g-matrix, we can write $g(\boldsymbol{\Psi}^{-1} + \mathbf{C}'\boldsymbol{\Sigma}_\varepsilon^{-1}\mathbf{C}) = \boldsymbol{\Psi}^{-1} + \mathbf{C}'\boldsymbol{\Sigma}_\varepsilon^{-1}\mathbf{C}$. We then replace $\mathbf{C}'\boldsymbol{\Sigma}_\varepsilon^{-1}\mathbf{C}$ by $g(\boldsymbol{\Psi}^{-1} + \mathbf{C}'\boldsymbol{\Sigma}_\varepsilon^{-1}\mathbf{C}) - \boldsymbol{\Psi}^{-1}$ in the right hand side of Equation (16) and confirm the equality. We conclude $\mathbf{U} = a^{-1}\boldsymbol{\Sigma}_\varepsilon^{-1}\mathbf{C}\mathbf{D}$ represents a solution.

F Proof of Theorem 1

We replace $\mathbf{B}_{2n}$ in the third part of Equation (8) from the main paper with its value given in Equation (12). We then obtain $\mathbf{A}_2$. We obtain $\mathbf{A}_1$ directly from the expression for $\mathbf{U}$. In order to determine A_0, we substitute the following expression for B_{0n} into the first part of Equation (8) from the main paper.

$$B_{0n} = (\mathbf{C} - \mathbf{B}_{1n}\mathbf{M}_n - \mathbf{B}_{2n}\mathbf{A}_1) E[\tilde{\eta}] - \mathbf{B}_{2n}(A_0 - \mathbf{A}_2 E[\tilde{z}])$$

To check that $\mathbf{A}_2$ is a regular matrix, we start with Equation (13) from the main paper. Note that matrices $\mathbf{C}$, $\mathbf{D}$, $\mathbf{Q}$ and $\mathbf{\Sigma}_\varepsilon$ are, by definition, regular matrices. Moreover, $\mathbf{CQC}' + \mathbf{\Sigma}_\varepsilon - \mathbf{V}_N$ is a positive definite matrix, thus regular. The positive-definiteness can be seen by noting that $\mathbf{CQC}' + \mathbf{\Sigma}_\varepsilon$ is the variance-covariance of a totally uninformed investor who does not even observe equilibrium prices. Matrix $\mathbf{V}_N$, on the other hand, is the variance-covariance matrix of the "average investor" who has some private information and observes prices.

G Symmetric and Complete Information

To get the full information CAPM in terms of prices, first subtract R times Equation (4) from the main paper from Equation (3) from the main paper and take expectations to get:

$$E\left[\tilde{P}^1\right] - RE\left[\tilde{P}^0\right] = (\mathbf{C} - R\mathbf{A_1})E\left[\tilde{\eta}\right] + R\mathbf{A}_2 E\left[\tilde{z}\right] - RA_0$$

Equation (11) from the main paper, Equation (12) from the main paper, and Equation (13) from the main paper enable us to write:

$$\begin{array}{rclcl} R\mathbf{A}_1 & = & (\mathbf{CQC}' + \mathbf{\Sigma}_\varepsilon - \mathbf{V}_N)(\mathbf{CDQC}')^{-1}\mathbf{CD} & = & \mathbf{C} \\ R\mathbf{A}_2 & = & a(\mathbf{CQC}' + \mathbf{\Sigma}_\varepsilon - \mathbf{V}_N)(\mathbf{CDQC}')^{-1}\mathbf{\Sigma}_\varepsilon & = & a\mathbf{\Sigma}_\varepsilon \\ RA_0 & = & (\mathbf{C} - R\mathbf{A}_1)E\left[\tilde{\eta}\right] + (R\mathbf{A}_2 - a\mathbf{V}_N)E\left[\tilde{z}\right] & = & 0 \end{array}$$

Therefore:

$$\begin{array}{rcl} E\left[\tilde{P}^1\right] - RE\left[\tilde{P}^0\right] & = & a\mathbf{\Sigma}_\varepsilon E\left[\tilde{z}\right] \\ & or & \\ E\left[\tilde{P}^0\right] & = & \frac{1}{R}\left(E\left[\tilde{P}^1\right] - a\mathbf{\Sigma}_\varepsilon E\left[\tilde{z}\right]\right) \end{array}$$

We can express the full info CAPM in terms of covariance which makes it more familiar to financial economists. When all investors are informed, they know the realization of $\tilde{\eta}$ is η. Therefore, $\tilde{P}^1 = \mathbf{C}\eta + \tilde{\varepsilon}$ and $Var\left[\tilde{P}^1\right] = \mathbf{\Sigma}_\varepsilon$:

$$\begin{array}{rclclcl} a\mathbf{\Sigma}_\varepsilon E\left[\tilde{z}\right] & = & a\,Var\left[\tilde{P}^1\right]E\left[\tilde{z}\right] & = & a\,Cov\left[\tilde{P}^1, \tilde{P}^1\right]E\left[\tilde{z}\right] & = & a\,Cov\left[\tilde{P}^1, \left(\tilde{P}^1\right)' E\left[\tilde{z}\right]\right] \\ & = & a\,Cov\left[\tilde{P}^1, \tilde{P}^1_m\right] & & & & \end{array}$$

Above, $\tilde{P}^1_m$ is the payoff of the market portfolio (the one that contains all the assets) divided by the number of investors (since $\tilde{z}$ has been defined as the supply per investor). As the supply is unknown by the agents in the market, we consider the expectations of the supply, rather than the supply itself. Using Equation (16) from the main paper and the above result gives the following expression for a single asset j:

$$E\left[\tilde{P}^1_j\right] - RE\left[\tilde{P}^0_j\right] = aCov\left[\tilde{P}^1_j, \tilde{P}^1_m\right]$$

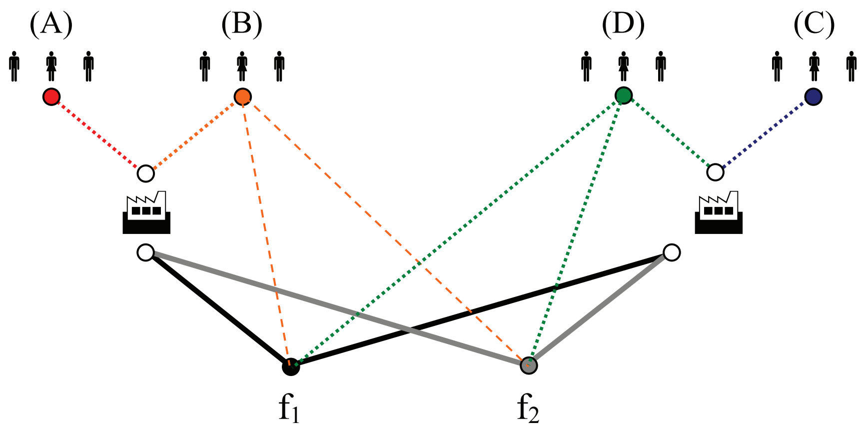
H Information Structures

The diagrams on the following pages depict worlds with four groups of investors labeled A, B, C, and D. There are four assets depicted by factories. There are two factors denoted "f_1" and "f_2". Solid lines from factors to assets, if they exist, indicate that payouts are determined by an underlying factor structure. Dashed lines from investor groups to either assets or to factors indicate information.

Information Structures Inspired by Albuquerque, Bauer, and Schneider (2009)

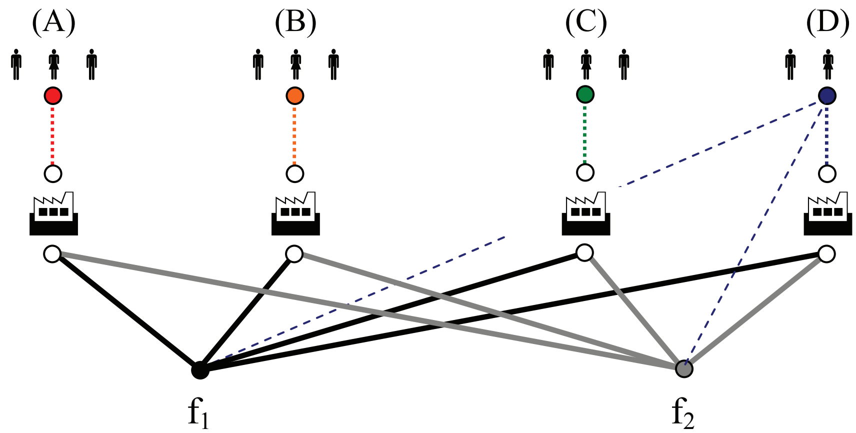
Panel A: Base Case

The diagram depicts an information structure inspired by the one in Albuquerque, Bauer, and Schneider (2009). Asset payoffs are assumed to be affected by two global factors denoted f_1 and f_2 (the original paper has a single global factor denoted "G"). Investor Groups A and B are assumed to be from the USA. Investor Groups C and D are assumed to be from the UK. Investors from each country are assumed to have asset-specific information about their "home" asset. Here, Group B and Group D have information about the global factors. Also, each of the four groups of investors has asset-specific information about two assets that are neither from the USA nor from the UK (assets 3 and 4 are not depicted for clarity, but have been included in this structure so that results can be compared with the other structures shown in Table 2 of our main paper).



Panel B: Albuquerque et al. v2

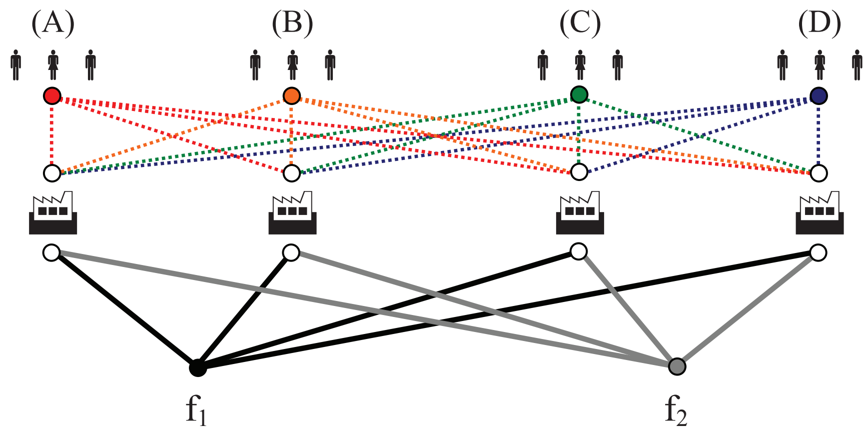
The diagram depicts an information structure inspired by Albuquerque, Bauer, and Schneiderr (2009). Asset payoffs are assumed to be affected by two global factors. Here, Group D has information about both global factors. Also, each of the four groups of investors has information about its "home" asset.



Information Structures Inspired by Kodres and Pritsker (2002)

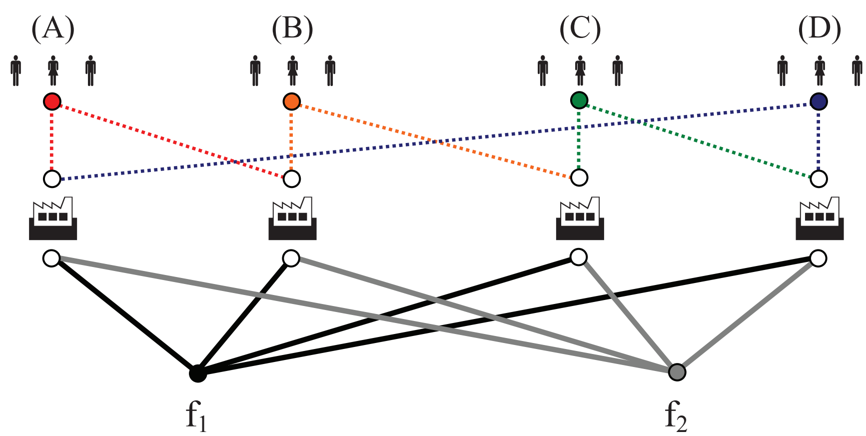
Panel A: Base Case

The diagram depicts an information structure consistent with the one in Kodres and Pritsker (2002). Asset payoffs are assumed to be affected by an underlying factor structure. No group of investors, however, has information about these underlying factors. Each group of investors has asset-specific information about each of the four assets.



Panel B: Kodres and Pritsker v.1

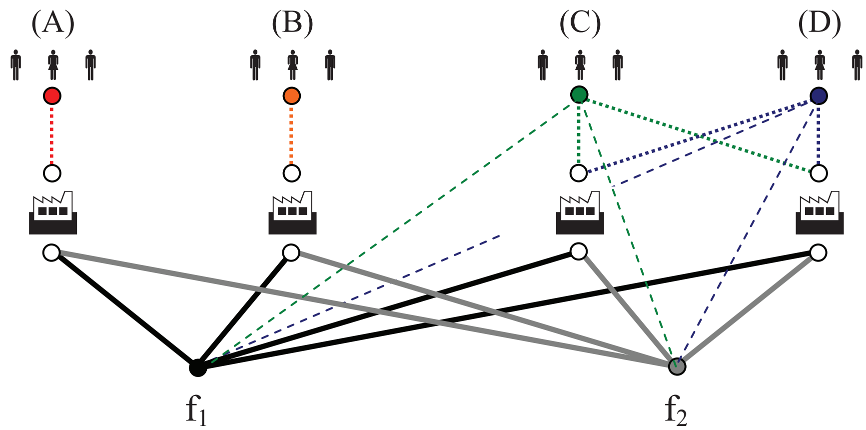
The diagram depicts an information structure inspired by Kodres and Pritsker (2002). Asset payoffs are assumed to be affected by an underlying factor structure. No group of investors, however, has information about these underlying factors. Each of the four groups of investors has asset-specific information about two of the assets.



Additional Information Structures

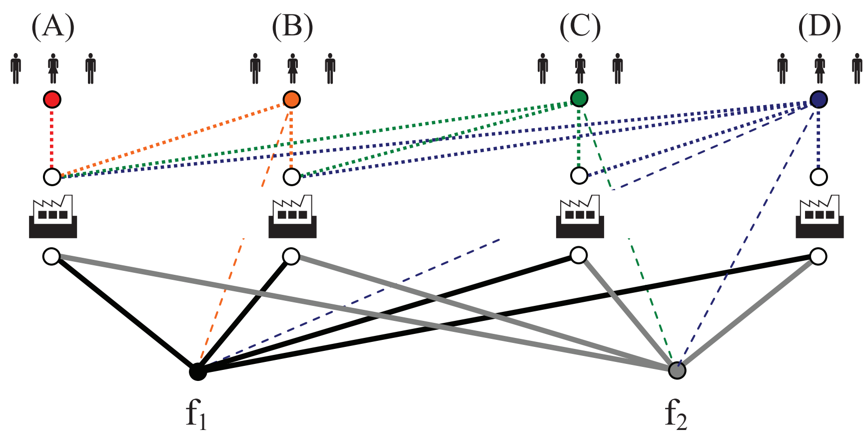
Panel A: Additional v.2

The figure depicts one of many additional information structures modeled in the current manuscript. In this diagram, investor groups C and D are essentially the same. Both know asset-specific information about assets 3 and 4. Both know information about factors f_1 and f_2.



Panel B: Additional v.3

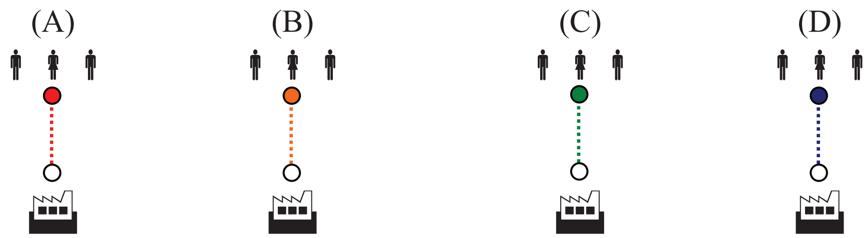
The figure depicts one of many additional information structures modeled in the current manuscript. There is an information hierarchy. Investor group A knows the least, Investor group B knows a bit more, and so on.



Traditional Home Bias Information Structure

Panel A: Traditional Home Bias

The diagram depicts the information structure in traditional home bias studies. Each group of investors has information about one of the assets (i.e., the home asset). There is no underlying factor structure which is similar to assumptions made in most basic studies of home bias.



I Notes on Numerical Solutions

The system in Equation (8) from the main paper represents a fixed point problem in a $2J^2 + JK + J$ Euclidian space. Such a system can be solved numerically for small values of J and K. The parameters used to generate Table 3 and Table 4 are as follows:

Investor groups	$N = 4$	$n = 1, \ldots, N$
Fraction of investors in economy	$\lambda_n = 0.25$	$n = 1, \ldots, N$
Names of groups in diagrams	A, B, C, D	
Assets	$J = 4$	$j = 1, \ldots, J$
Names of assets in diagrams	1, 2, 3, 4	
Common components	$K = 2$	
Names of common components	$\tilde{f}_1$ and $\tilde{f}_2$	The expected values of $\tilde{f}_1$ and $\tilde{f}_2$ are zero.
Factor loadings	$\mathbf{B}(1.0)$	All 4×2 elements equal one
	$\mathbf{B}(0.5)$	Used in Table 4
	$\mathbf{B}(1.5)$	Used in Table 4
Risk aversion coefficient	$a = 1$	
Riskfree rate	$r_f = 0$	$R = 1 + r_f = 1$
Per capital asset supply	$\tilde{z} \sim N(1, \mathbf{\Sigma}_z)$	$\tilde{z}$ is a J x 1 vector. $\mathbf{\Sigma}_z = I$
Residual uncertainties	$\tilde{\epsilon} \sim N(0, \mathbf{\Sigma}_\epsilon)$	$\tilde{\epsilon}$ is a J x 1 vector. $\mathbf{\Sigma}_\epsilon = I$
Asset-specific payoffs	$\tilde{\theta} \sim N(20, \mathbf{\Sigma}_\theta)$	$\tilde{\theta}$ is a J x 1 vector. $\mathbf{\Sigma}_\theta = I$

J Notes on Closed-Form Solutions

In order to analyze the relations between information structures, prices and holdings, we propose 20 different information structures. The structures are summarized in Table 2 in the main paper. As noted in the Table 2's final column, 14 of the 20 information structures allow for closed-form solutions. This means that, for the 14 structures:

- All the assumptions from Section 4.1 in the main paper are satisfied.
- We verify that, for a set of parameters, the term $(\mathbf{\Psi}^{-1} + \mathbf{C}'\mathbf{\Sigma}_{\epsilon}^{-1}\mathbf{C})$ is a "g-matrix" (as required by Lemma 1 in the main paper.)
- We also confirm that, for a set of parameters, our numerical and closed-form solutions give the same asset prices and holding values.

For this Internet Appendix, we choose to focus on one of the 14 structures—labled "ABS-Inspired v2". For this structure, there are 51 parameters values that must be specified including: a coefficient of risk aversion, the riskfree rate, the eight factor loadings, the expected supply shock for each of the four assets, the ten elements of the supply shocks' variance-covariance matrix, the expected value of the asset-specific component for each of the four assets, the ten elements of the asset-specific variance-covariance matrix, and the ten elements of the residual uncertainty's variance-covariance matrix.

There are infinite combinations of parameter values such that the assumptions of Lemma 1 are satisfied. However, not all combinations of parameter values result in the term $(\mathbf{\Psi}^{-1} + \mathbf{C}'\mathbf{\Sigma}_{\epsilon}^{-1}\mathbf{C})$ being a g-matrix. Therefore, we devise a method for choosing parameters that result in a g-matrix. We then verify that we do, in fact, have g-matrix. To accomplish our goal for the ABS-Inspired v2 structure, we propose a simpler methodology and a more general methodology.

J.1 A Simpler Method Applied to the ABS-Inspired v2 Structure

We start by defining the parameter values for this structure. We make some simplifying assumptions that reduce the number of free parameters from 51 to 6:

- There are four groups of investors. Each group is in equal numbers.
- There are four assets.

- There are two common components.
- The factor loading matrix is set to $\mathbf{B} = [\text{b1 b1; 1 1; 1 1; 1 1}]$. Note that $b1$ is a parameter that can be changed.
- The expected value of θ is 20 for each asset.
- For the $\mathbf{\Sigma}_\theta$ matrix, we first create $s_\theta = [\text{0.5 0 0 t1; 0 0.5 0 t2; 0 0 0.5 t3; 0 0 0 0.5}]$. Note that $t1$, $t2$, and $t3$ are variables. We then calculate $\mathbf{\Sigma}_\theta = s_\theta s_\theta'$
- The expected value of z is one for each asset.
- For the $\mathbf{\Sigma}_z$ matrix, we first create $s_z = [\text{0.25 0 0 z1; 0 0.25 0 z2; 0 0 0.25 z3; 0 0 0 0.25}]$. Note that $z1$, $z2$, and $z3$ are variables. We then calculate $\mathbf{\Sigma}_\mathbf{z} = s_z s_z'$
- We set $\mathbf{\Sigma}_\epsilon$ equal to the identity matrix.
- The risk aversion coefficient is one.

We next solve for $\mathbf{\Sigma}_\theta$ and $\mathbf{\Sigma}_z$ such that the term $(\mathbf{\Psi}^{-1} + \mathbf{C}'\mathbf{\Sigma}_\epsilon^{-1}\mathbf{C})$ is a g-matrix. We need the following equality to hold:

$$\sum_{n=1}^{4} \mathbf{D}_n(\mathbf{\Psi}^{-1} + \mathbf{C}'\mathbf{\Sigma}_\epsilon^{-1}\mathbf{C})\mathbf{D}_n = \mathbf{\Psi}^{-1} + \mathbf{C}'\mathbf{\Sigma}_\epsilon^{-1}\mathbf{C}$$

Solving for $\mathbf{\Sigma}_\theta$ and $\mathbf{\Sigma}_z$ involves a system of six linear equations with unknown variables $t1, t2, t3, z1, z2, z3$. The first part of our solution is:

$$\mathbf{\Sigma}_\theta = \begin{bmatrix} \frac{1}{4} + \frac{1}{16}b1^2 & \frac{1}{16}b1 & \frac{1}{16}b1 & -\frac{1}{8}b1 \\ \frac{1}{16}b1 & \frac{5}{16} & \frac{1}{16} & -\frac{1}{8} \\ \frac{1}{16}b1 & \frac{1}{16} & \frac{5}{16} & -\frac{1}{8} \\ -\frac{1}{8}b1 & -\frac{1}{8} & -\frac{1}{8} & \frac{1}{4} \end{bmatrix}$$

The second part of the solution is:

$$\mathbf{\Sigma}_z = \begin{bmatrix} \frac{1}{16} + \frac{1}{4}b1^2 & \frac{1}{4}b1 & \frac{1}{4}b1 & \frac{1}{8}b1 \\ \frac{1}{4}b1 & \frac{5}{16} & \frac{1}{4} & \frac{1}{8} \\ \frac{1}{4}b1 & \frac{1}{4} & \frac{5}{16} & \frac{1}{8} \\ \frac{1}{8}b1 & \frac{1}{8} & \frac{1}{8} & \frac{1}{16} \end{bmatrix}$$

As both the above solutions are functions of $b1$ we can generate infinitely many solutions—any one of which ensures $(\mathbf{\Psi}^{-1} + \mathbf{C}'\mathbf{\Sigma}_\epsilon^{-1}\mathbf{C})$ is a g-matrix.

J.2 A More General Method Applied to the ABS-Inspired v2 Structure

We highlight differences from the simpler methodology outlined above:

- The factor loading matrix is set to $\mathbf{B}$ = [b1 b1; b2 b2; b3 b3; b4 b4]. Note that $b1$, $b2$, $b3$, and $b4$ are all parameters that can be changed.
- For the $\mathbf{\Sigma}_\theta$ matrix we first create s_θ = [t0 0 0 t1; 0 t0 0 t2; 0 0 t0 t3; 0 0 0 t0]. Note that $t0$ is a parameter that can be changed and $t1$, $t2$, and $t3$ are variables. We then calculate $\mathbf{\Sigma}_\theta = s_\theta s_\theta'$
- For the $\mathbf{\Sigma}_z$ matrix we first create s_z = [z0 0 0 z1; 0 z0 0 z2; 0 0 z0 z3; 0 0 0 z0]. Note that $z0$ is a parameter that can be changed and $z1$, $z2$, and $z3$ are variables. We then calculate $\mathbf{\Sigma}_z = s_z s_z'$

Solving for $\mathbf{\Sigma}_\theta$ and $\mathbf{\Sigma}_z$ again involves a system of six linear equations with unknown variables $t1$, $t2$, $t3$, $z1$, $z2$, $z3$. The flexibility of this method stems from the fact that, while the simpler method has one parameter that can be changed, the more general method has six parameters that can be changed. Adjusting these six parameters allows one better target different values in $\mathbf{\Sigma}_\theta$ and $\mathbf{\Sigma}_z$ while still ensuring that $(\mathbf{\Psi}^{-1} + \mathbf{C}'\mathbf{\Sigma}_\epsilon^{-1}\mathbf{C})$ is a g-matrix.

References

Admati, A. R. (1985) A noisy rational expectations equilibrium for multi-asset securities markets, *Econometrica* **53(3)**, 629–658.

Albuquerque, R., Bauer, G. and Schneider, M. (2009) Global private information in international equity markets, *Journal of Financial Economics* **94(1)**, 18–46.

Gehrig, T. (1993) An information based explanation of the domestic bias in international equity investment, *Scandinavian Journal of Economics* **95(1)**, 97–109.

Kodres, L. E. and Pritsker, M. (2002) A rational expectations model of financial contagion, *Journal of Finance* **57(2)**, 769–799.

Van Nieuwerburgh, S. and Veldkamp, L. (2010) Information acquisition and under-diversification, *Review of Economic Studies* **77(2)**, 779–805.