Common Factors, Information, and Holdings Dispersion*

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Abstract. We derive closed-form solutions for asset prices and portfolio holdings when agents have asset-specific information and/or information about common components that affect many assets. Our solutions are general, encompass existing information structures, and are used to analyze new structures. A given investor's portfolio can exhibit highly disperse holdings—e.g., portfolio weights may vary significantly from market capitalization weights. Our model also generates large ranges of asset prices due to information asymmetries. We help explain why US investors (for example) may underweight German stocks (for example) on average, but overweight a particular German stock relative to its market capitalization weight.

1. Introduction

How do investors gauge the value of asset-specific information versus information about common components that affect many assets? How valuable is one's asset-specific information if other investors have common component information that is relevant for the asset in question? How do asset prices differ when one group of investors has all common-component information versus situations in which different groups have information about different common components? While the above three questions are focused on prices, one can ask related questions about investors' holdings of different assets. In particular, financial economists are interested in knowing which information structures lead different

investors to hold similar portfolios and which structures produce concentrated, disperse, and/or varied holdings.

Throughout this paper, we use the term "holdings dispersion" to indicate portfolio weights that vary significantly from benchmark weights. Market capitalization weights are a common benchmark. We know that the portfolio weights of all investors (in aggregate) equal market capitalization weights. A focus of this paper is to understand why a given investor over- or underweights a given stock relative to benchmark weights. In other words, one can think of holdings dispersion as a measure of "distance" from a benchmark portfolio.

This paper presents a generalized theory model that allows researchers to study tradeoffs between asset-specific information and common-component information (i.e., information that simultaneously affects two or more assets). The model's results can be applied to a wide range of studies in the field of financial economics. Examples of possible studies include, but are not limited to, three equity-related areas: 1) Cross-border equity holdings; 2) Mutual fund holding patterns within a given country (e.g., US mutual fund holdings of US equities); and 3) Employees' holdings of their own companies' equities. Topics such as cross-border holdings are well studied, while links between information and employees' holdings of their own companies' stocks have received less scrutiny in the academic literature.

Holdings dispersion is a salient characteristic of cross-border investing and the home-bias literature. Numerous studies have documented that investors tend to invest the majority of their portfolios in "home-country" assets.[1] Drilling down to study institutional holdings at the company-stock level, one quickly notices that some stocks are favored by foreign investors while other stocks are held almost exclusively by locals. Consider aggregate mutual fund positions in 467 German stocks as of 31-Dec-2002 (using data from Thomson Financial). From the perspective of the average German stock, foreign funds in the Thomson Financial database own 3.66% of the shares outstanding. For one quarter of the German stocks, this same group of funds holds less than 0.01% of the equity in aggregate. For the upper quarter of German stocks, these foreign funds hold at least 4.35% of the equity.[2] Foreign ownership dispersion of this magnitude is typical when looking at non-French mutual fund positions in French stocks, non-UK fund positions in UK stocks, and so on. The results of our theory model shed light on why cross-border ownership of equities may vary considerably across assets.

One contribution of the paper is to produce compact, closed-form solutions for asset prices and investor holdings in a world with multiple assets, multiple sources of information, and common components. Our expressions give insights into the tradeoffs facing an investor with asset-specific information—especially when other investors may have common-component information. In our framework, the effects of asset-specific vs. common-component information on portfolio holdings are not altogether obvious. For example, an investor may have high demand for a stock for which he has asset-specific information. He may also have high demand for the same stock if he has valuable private information about a stock with highly correlated payoffs. When considering common

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[1] Section 1.1 reviews related papers. Weights in home country stocks tend to be 80% to 90% even though home country stocks may constitute 5% to 30% of the world market portfolio.

[2] Note that measuring percentages of shares outstanding is a way to normalize holdings and is closely related to market capitalization weights. If a group of investors hold the same percentage of every stocks' shares outstanding—say 1%—the investors' portfolio weights equal market capitalization weights.

components, an investor may have high demand for a stock (even if he does not have asset-specific information) provided he has information about a common component that affects the stock's payoffs. Of course, having information about a common component is not sufficient to determine whether an investor has high demand for certain stocks. Stocks must load sufficiently on the common component (factor) to outweigh private, asset-specific information that other investors may have. Finally, and in a multi-asset setting, agents balance information about a given asset with a desire to diversify wealth across many assets. Therefore, high demand from one informed investor may be "over-run" by high demands from other investors who value a stock for hedging purposes.

Our solutions are more than simply compact. Existing papers have hypothesized about a single global factor for which some investors have information. Our paper offers three advancements from earlier works. First, we allow for multiple common components. Second, loadings can differ across components and stocks. Third, different investor groups may have information about different common components. These three aspects set our model apart from existing papers. Put simply, no one group of investors has an absolute informational advantage over other groups. A given group's informational advantage depends on tradeoffs between common-component and asset-specific information, access to common-component information, and stocks' loadings on the different common components in the economy.

While one can debate the value of closed-form solutions, they have (historically) been *de rigure* in economics. Our closed-form solutions for prices are especially useful in that they prompt us to solve explicitly for an "information discount factor" or "DF_{info}". The DF_{info} is defined as the difference between asset prices in a (possibly hypothetical) frictionless world and the prices that come out of our model.[3] Put differently, when many investors worry about adverse selection (due to others having asset-specific or common-component information) prices are low and expected returns are high. Our DF_{info} expression shows exactly the magnitude of the price discounts and why they exist.

Another contribution of our framework is that it produces solutions for a wide-range of information structures. Our model encompasses structures used in existing papers.[4] We also offer new information structures that financial economists can study. We believe one of this paper's main contributions is that our approach allows us to quantify and rank the severity of information-based frictions in a market.[5] In fact, many existing papers essentially quantify informational advantages with a binary variable (e.g., locals have firm-specific information while non-locals do not). Because our paper includes common components and allows stocks to load differently on the common components, informational advantages can vary subtly (continuously) across investors. The fact that stocks may have different (continuous) factor loadings leads investors with common component information to have different (continuous) informational advantages across stocks.

One information structure that we can model is the case of symmetric and complete information as in the traditional Capital Asset Pricing Model (also known as the "Frictionless CAPM" or "Full-Information CAPM"). Figure 1 depicts an equity market in which

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[3] Information asymmetry is the friction of focus in our paper. Therefore, when we discuss a "frictionless world", it is one in which agents have symmetric and complete information.
[4] We can model structures inspired by Albuquerque, Bauer, and Schneider (2009) and by Kodres and Pritsker (2002).
[5] Obtaining closed-form solutions requires us to make some assumptions. We evaluate the severity of the assumptions.

investors are partitioned into four groups. In this example, there are four assets and two common components in the economy. Panel A depicts the frictionless/full-info CAPM structure in which each investor group has all possible asset-specific information and all common-component information (information links are denoted with dotted/dashed lines).

[Insert Figure 1 About Here]

To contrast with the CAPM, Figure 1, Panel B depicts a different information structure.[6] This structure is one of the 20 different information structures that we model in Section 3. (this one happens to be called "Additional Structure v6"). In this structure, the investors clearly have different information. Investor Group A has asset-specific information about assets 1 and 2. Investor Group B has asset-specific information about asset 3. Neither Group A nor Group B has information about the common-components. Both Group C and Group D have asset-specific information about asset 4 and information about common components f_1 and f_2. After looking at Figure 1, Panel B, one can determine that investor Group C and Group D have the "most" information followed by Group A and then by Group B. Assuming the four assets have equal expected payoffs, it is difficult to assess price differences across assets. Which asset costs the most? Which one costs the least? One of our model's contributions is to quickly produce relevant quantities—such as an investor's holdings of each asset as well as each asset's equilibrium price.

Finally, we are able to produce large dispersions in portfolio holdings in a parsimonious manner and without relying on a large number of narrow assumptions.[7] Throughout the paper we emphasize two different types of holdings dispersion. We refer to "**absolute dispersion**" as a measure of how a population's holdings differ from the market portfolio. We refer to "**factor-based dispersion**" as the differences in holdings (across investors and/or stocks) that arise when the asset-specific components of information are equal. This type of dispersion is not directly tied to differences in asset-specific information. Instead, it is primarily driven by common-component information and factor loadings. The concept of *factor-based dispersion* represents a key contribution of our paper.

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[6] Readers can evisage a market with many, many listed stocks, numerous investor groups with different asset-specific information, and a number of common components. Supportive of such views, Chen, Roll, and Ross (1986) document nine macroeconomic risk factors that affect stock returns.

[7] A traditional assumption is that observed levels of home bias are driven by investors' local information advantages. However, such an assumption generally leads to low dispersion across <u>non-local</u> holdings (non-local stocks are held in similar <u>proportions</u> to a base (CAPM) model even though non-local holdings, as a group, are underweight.) To generate differences from CAPM weights for non-local stocks, one would need a long list of assumptions to explain the weight of German stocks in US investors' portfolios vs. the weight of French stocks in US investors' portfolios (as well as assumptions to explain the weight of French and US stocks in German investors' portfolios, and so on.) Instead of having many assumptions, our goal is to explain holdings dispersion as parsimoniously as possible. Considering multiple investors, multiple assets, and common-component information are all important for achieving this goal.

1.1 LITERATURE REVIEW

Our paper can be linked to a vast amount of previous work. The previous work can be categorized (loosely) into four streams. The first two streams are theory models: a) Multi-asset models about holdings dispersion and b) Average levels of holdings dispersion. The third and fourth streams are empirical and focus on analyzing holdings dispersion.

Theoretical models about holdings dispersion: Our paper is related to theoretical work on information structures, investor holdings, and risk premia. First, there is a clear link between our paper and Admati (1985). Internet Appendix B offers a detailed comparison of our proofs and those in the earlier model. All appendices are in an Internet Appendix which can be accessed via the authors' websites. A more recent paper, Easley and O'Hara (2004), presents a multi-asset model that focuses on the role of public and private signals in determining a firm's cost of capital. Private signals in their model are received only by a group of informed investors as in Grossman and Stiglitz (1980). In our model, it is possible for different groups of investors to have information about different groups of securities. In this way, our investors can be asymmetrically informed without introducing a strict information hierarchy.[8] Bacchetta and van Wincoop (2006) argue in favor of structures with a "[broad] dispersion of information."

In a paper similar in spirit to ours, Hughes, Liu, and Liu (2007) model two groups of investors. Each informed investor effectively observes a global signal "s" and these signals are perfectly correlated across investors. Unlike our paper, investors in the Hughes, Liu, and Liu (2007) paper cannot separate the asset-specific and global components. Additionally, information about the components is not differentially dispersed across investors. Similarly, Kodres and Pritsker (2002) offer a model that contains an underlying factor structure. However, there are no information asymmetries regarding the factors.

The paper by Albuquerque, Bauer, and Schneider (2009) or "ABS" is clearly the most related to our paper—though there are important differences. While our model is developed in a static setting, their paper is a dynamic asset pricing model with asymmetric information in the same vein as Brennan and Cao (1997). The payoff of a given stock is equal to the sum of three terms: a constant, a local component, and a single global factor. There are public and private signals about both the local component and the global factor. The ABS paper has the objective to understand the role of local and global information (both public and private) on the foreign holdings of US investors. Their paper, however, does not explain the differences across foreign stocks holdings. Put differently, the ABS paper does not try to answer questions such as: How can the global information explain why German stock A is preferred by US investors while German stock B is not? Complementing the earlier paper, and answering such questions, is precisely the contribution of our paper. Our model contains multiple global factors, each of which may be known by a different group of investors. While the Albuquerque, Bauer, and Schneider (2009) model considers that different investors may have different information about the common factor,

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[8] Our model incorporates one aspect of a number of models that endow all agents with small pieces of information about risky assets payoffs. For examples, see Grossman (1976), Hellwig (1980), and Admati (1985). Coval (1997) uses diffuse information in a manner similar to our paper. VanNieuwerburgh and Veldkamp (2009) study information acquisition and dynamic learning.

their model uses a single factor loading (equal to one) for all assets. Such an assumption implies that their model does not generate factor-based holding dispersion based solely on common-factor information, whereas our model is able to produce large differences in cross-border holdings (home bias) that can be tied directly to informational differences about common components. Our model offers new generalities because it allows for different common factor loadings—as well as information differences both across assets and across factors.

Recently, Dumas, Lewis, and Osambela (2017) proposed a model to study international portfolio choice when both domestic and foreign investors observe the same public signals but interpret them in a different way. In their model, the domestic investor is assumed to be better able to understand his own-country information. In our model, an investor who has information about a given global factor may have an information advantage about a foreign country's asset (relative to local investors) if this asset loads sufficiently to the global factor.

Finally, two published papers explore learning about categories (common components). Peng and Xiong (2006) allow a representative investor to learn about a stock's market-component, industry-component, or firm-specific component. While our paper is similar to theirs in this regard, we use heterogeneous investors and a rational expectations equilibrium, which differentiate our work. VanNieuwerburgh and Veldkamp (2010) jointly solve for the information and portfolio allocation problem. The authors mention on p.783 of their paper that correlated assets can be factored such that investors learn and invest in risk factors. Our model focuses on the tradeoff when investors simultaneously have asset-specific information and common component information.

Theoretical models about average levels of holdings differences: Studying ownership patterns from the perspective of a listed company is motivated by the well-known and extensive "home bias" literature. Information models relating asymmetries to home bias are well studied. As such, our paper speaks to this literature as well. Gehrig (1993) presents a related two-country model. Brennan and Cao (1997) study investment flows (changes in holdings) and information asymmetries. In their model, investors with less information (foreigners) update priors about future payoffs more heavily than investors with more information (locals). Van Nieuwerburgh and Veldkamp (2006) use a rational expectations equilibrium model to justify a persistent preference for home-country assets when investors initially have a small information advantage.

Empirical studies about holdings dispersion: Empirically, our paper is best viewed in terms of a line of research that studies cross-border ownership patterns from the perspective of a listed company. Some of the best known papers in this large literature include Kang and Stulz (1997) who look at foreign holdings of Japanese stocks. Dahlquist and Robertsson (2001) study relations between Swedish firm characteristics and foreign ownership. Covrig, Lau, and Ng (2006) conduct a cross-country analysis of fund manager preferences for stock characteristics. Finally, Ferreira and Matos (2008) document preferences of institutional investors.

Empirical studies about average levels of holdings dispersion: Like the theoretical literature, some empirical studies focus on average holdings differences across investor populations. There is a large literature that is primarily focused on home bias. French and Poterba (1991) document that American investors allocate about 84% of their wealth

in domestic stocks, even though the weight of the American stocks in the world market portfolio is only about 50% (an overweighing of 34% or 1.68X). Using 1997 data, Ahearne, Griever and Warnock (2004) show that 89.9% of US portfolio holdings are allocated to US stocks, even though these stocks comprise 48.3% of the world market portfolio (overweight by 41.6% or 1.86X). Chan, Covrig, and Ng (2005) show that the degree of home bias in other countries is generally greater than 30%.

2. Generalized Model and Solutions

Our framework considers I investors indexed $i = 1, \ldots, I$ who trade at date 0 and consume at date 1. Each agent i can invest his initial wealth, w_i^0, in a riskless asset and J risky assets indexed $j = 1, \ldots, J$. The riskless interest rate is denoted r_f and we define $R \equiv (1 + r_f)$. For simplicity, we normalize the price of the riskless asset to one. Each risky asset j pays a liquidating dividend $\tilde{P}_j^1$ at date 1. The vector of final payoffs $\tilde{P}^1 = (\tilde{P}_1^1, \ldots, \tilde{P}_J^1)'$ is generated by a K-factor linear process:

$$\tilde{P}^1 = \tilde{\theta} + \mathbf{B}\tilde{f} + \tilde{\varepsilon} \tag{1}$$

The vector $\tilde{\theta} = (\tilde{\theta}_1, \ldots, \tilde{\theta}_J)'$ is the asset-specific component of payoffs, the vector $\tilde{f} = (\tilde{f}_1, \ldots, \tilde{f}_K)'$ contains the K common components (factors), and $\mathbf{B}$ is a $J \times K$ matrix of factor loadings. The remaining part of each asset's final payoff, $\tilde{\varepsilon} = (\tilde{\varepsilon}_1, \ldots, \tilde{\varepsilon}_J)'$, is referred to as residual uncertainty. We assume that $\tilde{\theta}$, $\mathbf{B}\tilde{f}$, and $\tilde{\varepsilon}$ are jointly multivariate normal and independent. We further assume that $\tilde{f}$ and $\tilde{\varepsilon}$ have mean zero. For tractability, we assume that the covariance matrix of $\tilde{f}$ is the identity matrix. The covariance matrix of $\mathbf{B}\tilde{f}$ is $\mathbf{BB}'$. Finally, the covariance matrix of $\tilde{\varepsilon}$ is denoted $\boldsymbol{\Sigma}_\varepsilon$. Table 1 summarizes and describes all variables.

[Insert Table 1 About Here]

The per-capita supply of risky assets is defined as the realization of a random vector $\tilde{z}$. The vector $\tilde{z}$ is independent and jointly normally distributed along with the other variables in the model and has a covariance matrix denoted $\boldsymbol{\Sigma}_z$. The assumption of random net supply is standard in rational expectations models. As Easley and O'Hara (2004) write, "one theoretical interpretation is that it approximates noise trading in the market. A more practical example of this concept is portfolio managers' current switch toward using float-based indices from shares-outstanding indices." In order to ensure the existence and uniqueness of the date 0 equilibrium price vector, $\tilde{P}^0$, we assume that $\boldsymbol{\Sigma}_\varepsilon$, $\boldsymbol{\Sigma}_\theta$, and $\boldsymbol{\Sigma}_z$ are regular matrices.

We assume all agents have an exponential utility function: $U(\tilde{w}_i^1) = -e^{-a\tilde{w}_i^1}$, where $\tilde{w}_i^1$ is the wealth of investor i on date 1. The utility function has a constant absolute risk aversion with coefficient $a > 0$, which is the same for all agents. The choice of utility functions is also common in rational expectations equilibrium models and ensures that an investor's demand for the risky asset is independent of his initial wealth. Let X_i be investor i's column vector risky-asset holdings. Investor i's final wealth is:

$$\tilde{w}_i^1 = w_i^0 R + X_i'(\tilde{P}^1 - R\tilde{P}^0) \tag{2}$$

2.1 INVESTORS' INFORMATION AND ADDITIONAL NOTATION IN OUR MODEL

We partition the I investors in our model into N non-overlapping groups labeled $n = 1, \ldots, N$. Each group of investors represents a fraction, λ_n, of the total number of investors (I) in the market such that $\sum_{n=1}^{N} \lambda_n = 1$.

In our model, investors belonging to the same group n possess the same private information (for asset-specific components and for common components), they face the same optimization problem, and they optimally choose identical portfolios. In this sense they can be said to be identical. We therefore use the following terms interchangeably (and a bit loosely): "investor i from group n", "investor group n", and "investor n". Similar to a Grossman and Stiglitz (1980) framework, we say investor n has asset-specific information about asset j if the investor knows the realization of θ_j. We say investor n has information about common-component k if the investor knows the realization of f_k. To simplify notation, we write the payoffs of the risky assets as:

$$\tilde{P}^1 = \mathbf{C}\tilde{\eta} + \tilde{\varepsilon} \tag{3}$$

Where, $\tilde{\eta} = (\tilde{\theta}'\ \tilde{f}')'$ is a $J + K$ column vector and $\mathbf{C}$ is a $J \times (J + K)$ block-diagonal matrix consisting of a $J \times J$ identity matrix, $\mathbf{I}_J$, and the matrix $\mathbf{B}$. The variance-covariance matrix of $\tilde{\eta}$ is $\mathbf{Q} = \begin{pmatrix} \mathbf{\Sigma}_\theta & 0 \\ 0 & \mathbf{I}_K \end{pmatrix}$ where $\mathbf{I}_K$ is the identity matrix of order K.

Definition 1. For each investor n, we define the diagonal matrix $\mathbf{D}_n$ of order $J + K$. Diagonal elements $1, \ldots, J$ in $\mathbf{D}_n$ correspond to asset-specific components. Diagonal elements $J + 1, \ldots, J + K$ in $\mathbf{D}_n$ correspond to common components. We set $\mathbf{D}_n(\cdot, \cdot) = 1$ if investor n knows the realization of the associated random variable in $\tilde{\eta}$ and $\mathbf{D}_n(\cdot, \cdot) = 0$ otherwise.

Definition 2. We define $\mathbf{D} \equiv \sum_{n=1}^{N} \lambda_n \mathbf{D}_n$. The matrix $\mathbf{D}$ plays an important role in our model as each element on the main diagonal represents the proportion of investors who know the realization of the corresponding random variable in the vector $\tilde{\eta}$.

Definition 3. For each investor group n, the matrix $\mathbf{M}_n$ is obtained by eliminating all the null rows of $\mathbf{D}_n$. Consequently, the number of rows of $\mathbf{M}_n$ is equal to $J_n + K_n$, which represents the number of asset-specific and common components about which investor n is informed. If investor n does not receive any private information, $\mathbf{D}_n$ becomes the null matrix and $\mathbf{M}_n$ cannot be defined. It is straightforward that $\mathbf{M}_n'\mathbf{M}_n = \mathbf{D}_n$ and $\mathbf{M}_n\mathbf{M}_n' = \mathbf{I}_{J_n + K_n}$, where $\mathbf{I}_{J_n + K_n}$ is the identity matrix of order $J_n + K_n$.

Under these definitions, the private information received by investor n consists of the realization of the random vector $\mathbf{M}_n\tilde{\eta}$. As is typical in a REE framework, equilibrium prices also reveal some information to investors beyond the investors' own private information. Consequently, each investor n maximizes his expected utility of consumption conditional on the realization of his private information and on the observation of the public information in the form of prices at date 0.

2.2 EQUILIBRIUM PRICES AND HOLDINGS

We seek solutions for prices and holdings at date 0 within the class of functions that are linear in our information variable $\tilde{\eta}$ and supply variable $\tilde{z}$. The form of the solution implies that investors assume prices are a linear function of private signals and noise. In equilibrium, this hypothesis is verified. The date 0 price vector is:

$$\tilde{P}^0 = A_0 + \mathbf{A}_1 \tilde{\eta} - \mathbf{A}_2 \tilde{z} \tag{4}$$

where A_0 is a $J \times 1$ vector, $\mathbf{A}_1$ is a $J \times (J + K)$ matrix, and $\mathbf{A}_2$ is a $J \times J$ matrix. We suppose that $\mathbf{A}_2$ is regular. Under these assumptions, investor n's demand is:

$$\tilde{X}_n = a^{-1} \mathbf{V}_n^{-1} \left(E_n \left[\tilde{P}^1 \right] - R \tilde{P}^0 \right) \tag{5}$$

Equation (5) gives an expression for agent n's holdings at date 0—please see Internet Appendix C for additional details. All appendices are in an Internet Appendix which can be accessed via the authors' websites. The expression $E_n[\tilde{P}^1] = E[\tilde{P}^1 | \mathbf{M}_n \tilde{\eta}, \tilde{P}^0]$ gives the expected prices of the risky assets at date 1 from investor n's point of view (i.e., conditional on his information set). $\mathbf{V}_n = Var[\tilde{P}^1 | \mathbf{M}_n \tilde{\eta}, \tilde{P}^0]$ represents the conditional variance of $\tilde{P}^1$ from investor n's point of view. By equating the supply and the aggregate demand of the N groups of investors, $\left(\sum_{n=1}^{N} \lambda_n \tilde{X}_n = \tilde{z} \right)$, it follows that

$$\sum_{n=1}^{N} \lambda_n \mathbf{V}_n^{-1} \left(E_n \left[\tilde{P}^1 \right] - R \tilde{P}^0 \right) - a\tilde{z} = 0 \tag{6}$$

Joint normality implies that the distribution of prices, conditional on investor n's private and public information, is also multi-variate normal with the following expectation:

$$E_n \left[\tilde{P}^1 \right] = E \left[\tilde{P}^1 | \mathbf{M}_n \tilde{\eta}, \tilde{P}^0 \right]$$
$$= B_{0n} + \mathbf{B}_{1n} \mathbf{M}_n \tilde{\eta} + \mathbf{B}_{2n} \tilde{P}^0 \tag{7}$$

where the dimension of B_{0n} is $J \times 1$, $\mathbf{B}_{1n}$ is $J \times (J_n + K_n)$, and $\mathbf{B}_{2n}$ is $J \times J$. Equations (4), (6), and (7) imply the system to be solved is (please see Internet Appendix D):

$$a\mathbf{A}_2^{-1} A_0 = \sum_{n=1}^{N} \lambda_n \mathbf{V}_n^{-1} B_{0n}$$

$$a\mathbf{A}_2^{-1} \mathbf{A}_1 = \sum_{n=1}^{N} \lambda_n \mathbf{V}_n^{-1} \mathbf{B}_{1n} \mathbf{M}_n \tag{8}$$

$$a\mathbf{A}_2^{-1} = \sum_{n=1}^{N} \lambda_n \mathbf{V}_n^{-1} \left(R\mathbf{I}_J - \mathbf{B}_{2n} \right)$$

We conclude this section with four notes. First, Equation (4) and the system in Equation (8) give equilibrium prices. Equation (5) gives equilibrium holdings of investors in group n. Second, to understand the relations between information variables, prices, and holdings we begin the next section by introducing 20 information structures. These information structures consider which investor groups have which asset-specific and common-component pieces of information. Third, we numerically solve for equilibrium prices and holdings. As shown in Internet Appendix D, the matrices $\mathbf{B}_{1n}$, $\mathbf{B}_{2n}$ and $\mathbf{V}_n$ can be written

as functions of the matrices $\mathbf{A}_1$ and $\mathbf{A}_2$. The system in Equation(s) (8) represents a fixed point problem in a $2J^2 + JK + J$ Euclidian space. Such a system can be solved numerically for small values of J and K. Fourth, we produce closed-form solutions for equilibrium prices and holdings. The assumptions needed to produce the closed-form solutions are not overly restrictive and closed-form solutions are available for 14 of the 20 structures.

3. Information Structures and Numerical Solutions

3.1 INFORMATION STRUCTURES

To understand the relations between information variables, prices, and holdings we turn to analyzing 20 different information structures. In each structure, we consider four, non-overlapping groups of investors. Each group of investors exists in equal numbers and the four groups are denoted A, B, C, and D. There are four assets denoted 1, 2, 3, and 4. Finally, there are two common-components denoted f_1 and f_2. We use two common components for parsimony and to differentiate our paper from models that only have a single global factor. In reality, our model allows for multiple common components and we could have shown structures with three or more factors (and five or more assets). We note that different information structures correspond to investor groups having different combinations of asset-specific and/or common-component information.

Table 2 presents the 20 information structures. The first structure represents a frictionless world in which each investor group knows all possible asset-specific and common-component information. Reading line labeled "Full-Info CAPM", we see that Group A has asset-specific information about assets 1, 2, 3, and 4 as well as information about common components f_1 and f_2. Groups B, C, and D have the same (full) information. Figure 1, Panel A graphically depicts this symmetric and complete information structure (i.e., the frictionless/full-information CAPM world). Figure 1 shows the four investor groups (labeled A to D) as well as the four assets (depicted as factories). Dotted and dashed lines indicate which investor groups have which pieces of information.

[Insert Table 2 About Here]

Below the line labeled "Full-Info CAPM", Table 2's next two lines show other symmetric information structures. By symmetric, we mean each of the four investor groups has the same information. No one group is at an advantage or disadvantage. Note that no group has complete information.

Below the symmetric cases, there are information structures that are inspired by two recently published papers. In Albuquerque, Bauer, and Schneider (2009) there is a single global factor called G. Using our framework to model the ABS (2009) results as faithfully as possible, we assume there are investors from two countries—see the Table 2 structure titled "ABS-Inspired (base)". Groups A and B are from the USA, while Groups C and D are from the UK. The USA investors have asset-specific information about the USA stock (1) while the UK investors have asset-specific information about the UK stock (2). All investors have asset-specific information for the two stocks from outside the USA and UK (stocks 3 and 4). Within each country, there are some investors who know only local (asset-specific) information (Groups A and C) and some investors who also have information about the global factors (Groups B and D). Note, there is a diagram of this structure in Internet Appendix H.

As shown in Table 2, we also consider five other structures in the spirit of Albuquerque, Bauer, and Schneider (2009) and called "$v1$", "$v2$", through "$v5$". In these structures, there are two global factors. In structures $v1$ and $v2$, Group D has information about both global factors. In addition to the ABS-Inspired (base) structure, Internet Appendix H also depicts the ABS-Inspired $v2$ structure.

Kodres and Pritsker (2002) allow asset prices to be determined by an underlying factor structure, but no group of investors has information about the factors. We model three structures inspired by the Kodres and Pritsker (2002) paper—the first two of which are pictorially shown in Internet Appendix H.

Finally, we consider eight additional structures. In the structure labeled Additional $v1$, Groups B, C and D have exactly the same information sets. In the structure labeled Additional $v3$, the amount of information increases as one moves from Groups A to B to C to D. Group D has asset-specific information for assets 1, 2, 3, and 4 as well as information about both f_1 and f_2. Internet Appendix H shows the second and third of these eight additional structures. Figure 1, Panel B shows pictorially the "Additional $v6$" structure. The final column in Table 2 indicates whether or not closed-form solutions are available for each structure. We will discuss closed-form solutions in Section 4.

3.2 NUMERICAL SOLUTIONS

This section numerically analyzes relations between information structures, equilibrium asset prices, and equilibrium holdings. We can vary which investor groups have different pieces of information while leaving the model parameters $\left\{ r_f, a, \lambda_1, \ldots, \lambda_N, \mathbf{B}, \mathbf{\Sigma}_\varepsilon, \overline{\theta}, \mathbf{\Sigma}_\theta, \overline{z}, \mathbf{\Sigma}_z \right\}$ constant. Rather than have conclusions depend on realizations of random variables, we study "ex-ante" prices and holdings by taking expectations over $\{\tilde{\eta}, \tilde{\varepsilon}, \tilde{z}\}$, the three random variables in the model.[9]

The model's parameters are kept simple in the numerical analysis so as to focus on relations between prices and information structures. The expected value of $\tilde{\theta}$ is 20 for each asset and $\mathbf{\Sigma}_\theta$ is the identity matrix. $\mathbf{\Sigma}_\epsilon$ is the identity matrix. The riskfree rate is zero. Investors are evenly distributed across groups ($\lambda_n = 0.25$ for n=1, ..., 4). The expected value of $\tilde{z}$ is one for each asset and $\mathbf{\Sigma}_z$ is the identity matrix. These assumptions imply that there are four shares per capita of each asset in expectation. There are a total of eight factor loadings in our example (four assets times two common components). We set all elements in the 4×2 factor loading matrix ($\mathbf{B}$) equal to one. The risk aversion coefficient is also one.

[Insert Table 3 About Here]

To efficiently display results from the numerical analysis, Table 3 reports the aggregate market capitalization associated with each of the 20 information structures. The aggregate market cap (MC\$) is the sum of prices for assets 1, 2, 3, and 4. The full-info CAPM world, not surprisingly, results in the highest prices and has a total market capitalization of

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[9] As noted, our approach involves taking expectations over random variables. An alternative methodology involves drawing a set of random variables $\{\tilde{\eta}, \tilde{\varepsilon}, \tilde{z}\}$ and calculating prices and holdings at date 0. Repeated draws of the random variables converge to the same expected values as the number of draws goes to infinity. Note that our methodology solves for prices and holdings before agents receive private information. As such, solutions are sometimes referred to as *ex-ante*. Parameters are given in Internet Appendix I.

$304.00. As information frictions become more severe, payoffs become relatively more risky, and prices fall. In the structure "Kodres & Pritsker v2" each investor group has a single piece of asset-specific information and no group has common-component information. The total market capitalization is $166.49, which is $137.51 below the full-info CAPM value (the $137.51 is called the "price discount"). The "degree of frictions" for this structure is 45%—as indicated in the third column of numbers—and is calculated as (1 - $166.49 ÷ $304 = 0.45). Note that a price discount from the full-info CAPM-price level represents one way to measure aggregate informational frictions in a market. Table 3 shows that, for our 20 structures, the degrees of frictions range from 0% to 45%.

Although not shown in Table 3, we note there is also a symmetric and frictionless CAPM in which no group of investors has any private information. We refer to such a structure as the "No-Information CAPM". This structure produces a total market capitalization of $160.00, which corresponds to a 47% price discount (1 - $160 ÷ $304 = 0.47). Either the full-info CAPM or the no-info CAPM can serve as a baseline by which to measure the effects of different information structures on equilibrium prices. If the latter were used, additional bits of information would lead to higher and higher prices.

We also numerically analyze holdings. In each of the 20 information structures, we have four groups of investors who hold the stocks of the four assets. To efficiently display holdings results, we introduce a measure of aggregate holdings dispersion. For each investor-asset pair, we calculate the difference between the asset's weight in a given investor's portfolio and the same asset's weight in the market portfolio. The difference can be thought of as an "error" in this example. We then calculate the root mean squared error (RMSE) across all investor-asset combinations. The RMSE provides a single aggregate measure that allows us to quantify how disperse holdings are relative to holding the market portfolio. We refer to this measure of dispersion as an "absolute" measure as it is based on a comparison to weights of the overall market portfolio. Table 4 shows the aggregate holdings dispersion measure (RMSE) for each of the 20 information structures.

[Insert Table 4 About Here]

For the holdings analysis, we continue to use the same parameters as mentioned above with one addition. For each of the 20 information structures, we vary the factor loadings matrix ($\mathbf{B}$) and consider three different values. We first set all the elements in the 4×2 factor loading matrix equal to 0.5 and record the total RMSE for each structure. We then repeat the analysis with the factor loadings equal 1.0 and then again for the loadings equal 1.5.

Start by looking at the middle column of Table 4, $\mathbf{B}(1.0)$. There is no holdings dispersion in the full-info CAPM — all investor groups have the same holdings and their portfolios are equal to the market portfolio. There is also no holdings dispersion for the two other symmetric structures. Next, look at the Kodres & Pritsker-inspired structures. The Kodres & Pritsker (base) structure is symmetric and there is no holdings dispersion. In the structure Kodres & Pritsker v1, each investor group has asset-specific information about only two of the four assets. Each group over-weights two assets and under-weights two assets leading each groups' holdings to differ from the market portfolio's weights. The net result is a RMSE, or holdings dispersion measure, of 0.079. Scanning the middle column, we see dispersion measures varying from 0.000 for the symmetric structures to 0.120 for the ABS-Inspired v2 structure.

The ABS-Inspired v2 structure has a rather extreme concentration of information. Investor Groups A, B, and C only posses asset-specific information about assets 1, 2, and

3 respectively. Investor Group D has asset-specific information about asset 4 as well as the common component information about both f_1 and f_2. The result is a 0.120 measure of holdings dispersion as shown in the middle column of Table 4.

For a given structure, Table 4 helps reinforce the concept of *factor-based dispersion*. Looking at the row labled "ABS-Inspired v2", we see the dispersion measure varies as the factor loadings change. When factor loadings are low, $\mathbf{B}(0.5)$, each investor group has an informational advantage about the payoffs of one of the assets and the common-component information is not very valuable. Investor Group D's information about the two common components becomes less valuable as the factor loadings move towards zero. As such the dispersion measure is 0.088 and considerable less than the 0.120 discussed above. When factor loadings are high in this structure, $\mathbf{B}(1.5)$, we see a 0.168 dispersion measure.

4. Closed-Form Solutions for Prices and Holdings

To get closed-form solutions for prices and holdings, we make certain (rather weak) assumptions about which groups have which information. This section first describes the informational assumptions. Second, we present the closed-form solutions. Third and fourth, we analyze equilibrium expressions for prices and holdings. Thanks to the closed-form solutions, we are able to offer analyses and detailed discussions about equilibrium prices and investors holdings. Such analyses are more transparent when inspecting equations than when looking at numerical solutions. Table 2's final column indicates that there are closed-form solutions available for 14 of the 20 structures. Fifth, we verify that solutions are consistent with the assumption made in this section. Sixth, we briefly discuss the restrictiveness of the assumptions in Section 4.5 and conclude the assumptions are not restrictive—we can obtain closed-form solutions for a wide range of information structures and markets.

4.1 ASSUMPTIONS ABOUT INFORMATION STRUCTURES

As mentioned in Admati (1985), rational expectations models show "enormous complexity". In order to solve a model and prove a closed-form solution in our rich and challenging setting, we need to make some assumptions about the information structures. These assumptions concern the way asset specific information and common component information is possessed by investors. One role of the assumptions is help to partition investors into groups, such that, within a given group, information is homogeneous across investors. Across groups, information is heterogeneous.

Asset-Specific Information: The J securities are partitioned into N non-overlapping groups.[10] We define the set of all assets as S. The set of assets in group n contains J_n risky assets and is denoted S_n. Thus, $\bigcup_{n=1}^{N} S_n = S$ and $\forall (n_a, n_b),\ n_a \neq n_b,\ S_{n_a} \cap S_{n_b} = \oslash$.

A single investor i in group n knows the realization of the asset-specific component, θ_j, of each asset j in the set S_n. For any asset j not in S_n, investor i only knows the distribution of $\tilde{\theta}_j$ but he does not know its realization. We assume there is an equal number (N) of securities groups and investors groups to ensure that each security has at

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[10] Note that Section 2.1 has already partitioned <u>investors</u> into N non-overlapping groups.

least one investor with asset-specific information.

Common Component Information: We assign each of the K common factors to one of N groups denoted F_n, with $n = 1, \ldots, N$. The set F_n contains K_n common components and $0 \leq K_n \leq K$. An investor i in group n knows the realization of each common component $\tilde{f}_k$ in the set F_n. For any component not in F_n, the investor knows the distribution of $\tilde{f}_k$, but not its realization. For tractability purposes of the model, we assume that two groups of investors do not have information about the same common component.[11]

We summarize the roles of the above mentioned assumptions and groups as follows: Our groups translate into the existence of representative investors (one for each investor group). These representative agents (who make up the market) all have different information about asset specific components and common components. The role of considering these representative investors is to make the mathematical proofs possible and obtain a closed-form solution.

4.2 CLOSED-FORM SOLUTIONS

To obtain a closed-form solution for $\tilde{P}^0$, we define the matrix $\mathbf{U} \equiv \mathbf{A}_2^{-1} \mathbf{A}_1$. We also introduce the function $g(\mathbf{G}) = \sum_{n=1}^{N} \mathbf{D}_n \mathbf{G} \mathbf{D}_n$, where $\mathbf{G}$ is a matrix of order $J + K$.

Definition 4. We define a "g-matrix" to be any square matrix $\mathbf{G}$ of order $J + K$ which satisfies $g(\mathbf{G}) = \mathbf{G}$.

We define $\mathbf{\Psi} \equiv Var\left[\tilde{\eta} | \tilde{P}^0\right]$ i.e., the variance-covariance matrix of $\tilde{\eta}$ conditional on observing the equilibrium price vector at date 0. The matrix $\mathbf{\Psi}$ is endogenously defined and represents the variance of $\tilde{\eta}$ from the point of view of an investor who does not possess any private information but only observes the equilibrium price vector. The following lemma gives an analytical solution for $\mathbf{U}$.

Lemma 1. *If $(\mathbf{\Psi}^{-1} + \mathbf{C}'\mathbf{\Sigma}_\varepsilon^{-1}\mathbf{C})$ is a g-matrix, then the closed-form solution for $\mathbf{U}$ is:*

$$\mathbf{U} = a^{-1}\mathbf{\Sigma}_\varepsilon^{-1}\mathbf{C}\mathbf{D} \tag{9}$$

Proof: *See Internet Appendix E. Having groups of investors with homogeneous information inside the group and heterogeneous information across groups allows us to find a closed form solution for our matrix $\mathbf{U}$. The six key properties exhibited in Internet Appendix E are true under the assumptions described in Subsection 4.1.*

For the particular case of Lemma 1, $\mathbf{U}$ is not a function of the coefficients B_{0n}, $\mathbf{B}_{1n}$, and $\mathbf{B}_{2n}$. Therefore, to determine A_0, $\mathbf{A}_1$, and $\mathbf{A}_2$, we must first compute the matrix $\mathbf{\Psi}$ as a function of $\mathbf{U}$. In this way, the variance-covariance matrix of any investor group, $\mathbf{V}_n$, can be written as a function of $\mathbf{\Psi}$:

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[11] If the number of common components is smaller than the number of investor groups, then $K < N$, some F_n sets will not contain any common components ($K_n = 0$), and the corresponding investor group will not be informed about any of the common components. Note that $K < N$ in Figure 1, Panel B.

$$\mathbf{V}_n = \boldsymbol{\Sigma}_\varepsilon + \mathbf{C}\boldsymbol{\Psi}\mathbf{C}' - \mathbf{C}\boldsymbol{\Psi}\mathbf{M}_n'\boldsymbol{\Psi}_n^{-1}\mathbf{M}_n\boldsymbol{\Psi}\mathbf{C}' \qquad (10)$$

Where $\boldsymbol{\Psi}_n = \mathbf{M}_n\boldsymbol{\Psi}\mathbf{M}_n'$. Also, $\boldsymbol{\Psi} = \mathbf{Q} - \mathbf{Q}\mathbf{U}'\mathbf{M}^{-1}\mathbf{U}\mathbf{Q}$ and $\mathbf{M} = \mathbf{U}\mathbf{Q}\mathbf{U}' + \boldsymbol{\Sigma}_z$. The following theorem gives a closed-form solution for the equilibrium price vector at date 0.

Theorem 1. *Under the conditions of Lemma (1), there exists a unique closed-form solution for Equation (6) within the class of linear functions of $\tilde{\eta}$ and $\tilde{z}$. The solution can be written as, $\tilde{P}^0 = A_0 + \mathbf{A}_1\tilde{\eta} - \mathbf{A}_2\tilde{z}$, where $\mathbf{A}_2$ is a regular matrix and:*

$$A_0 = \frac{1}{R}\Big(\big(\mathbf{C} - R\mathbf{A}_1\big)E[\tilde{\eta}] + \big(R\mathbf{A}_2 - a\mathbf{V}_N\big)E[\tilde{z}]\Big) \qquad (11)$$

$$\mathbf{A}_1 = \frac{1}{R}\big(\mathbf{C}\mathbf{Q}\mathbf{C}' + \boldsymbol{\Sigma}_\varepsilon - \mathbf{V}_N\big)\big(\mathbf{C}\mathbf{D}\mathbf{Q}\mathbf{C}'\big)^{-1}\mathbf{C}\mathbf{D} \qquad (12)$$

$$\mathbf{A}_2 = \frac{1}{R}a\big(\mathbf{C}\mathbf{Q}\mathbf{C}' + \boldsymbol{\Sigma}_\varepsilon - \mathbf{V}_N\big)\big(\mathbf{C}\mathbf{D}\mathbf{Q}\mathbf{C}'\big)^{-1}\boldsymbol{\Sigma}_\varepsilon \qquad (13)$$

The matrix $\mathbf{V}_N = (\sum_{n=1}^{N}\lambda_n\mathbf{V}_n^{-1})^{-1}$ represents the variance-covariance matrix of $\tilde{P}^1$ for the "average" investor in the market. The precision matrix $\mathbf{V}_N^{-1}$ equals the weighted mean of each group's precisions where the weights are proportional to the number of agents in each group. From Equation (10), it is straightforward to show that $\mathbf{V}_N$ can be written as:

$$\mathbf{V}_N = (\boldsymbol{\Sigma}_\varepsilon + \mathbf{C}\boldsymbol{\Psi}\mathbf{C}')(\mathbf{I}_J + \boldsymbol{\Sigma}_\varepsilon^{-1}\mathbf{C}\mathbf{D}\boldsymbol{\Psi}\mathbf{C}')^{-1} \qquad (14)$$

Proof: *See Internet Appendix F.*

To conclude, we provide closed-form solutions for prices and holdings at date 0. The solution for prices takes the form shown in Equation (4) with constant values shown in (11), (12), and (13). The form of investor Group n's holdings is shown in Equation (5).

4.3 ANALYSIS OF EQUILIBRIUM ASSET PRICES

We analytically study relations between information structures, parameter values, and equilibrium asset prices. This approach is the closed-form version of the numerical "ex-ante" pricing mentioned in Footnote 9. Note that, for a given information structure, there are typically infinitely many parameter combinations that are consistent with obtaining a closed-form solution. For the 14 structures that allow for closed-form solutions (see Table 2), we verify that, given a set of parameters, the term $(\boldsymbol{\Psi}^{-1} + \mathbf{C}'\boldsymbol{\Sigma}_\varepsilon^{-1}\mathbf{C})$ is in fact a g-matrix and all the assumptions from Section (4.1) are met. Internet appendix J has additional notes on the closed-form solutions.

General Model with Disperse Information: Rearranging Equation (6) gives a general expression for prices at date 0. Equation (15) below shows that asset prices at date 0 are less than the value of expected future payoffs.[12] The total price discount (risk premium) is given by the expression $a\mathbf{V}_N E[\tilde{z}]$. The price discount depends on risk aversion (a) and

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[12] Assuming assets are expected to be in positive net supply ($E[\tilde{z}] > 0$) and agents are risk averse ($a > 0$).

the market's "average" uncertainty about future payoffs ($\mathbf{V}_N$).

$$E\left[\tilde{P}^0\right] = \frac{1}{R}\left(E\left[\tilde{P}^1\right] - a\mathbf{V}_N E\left[\tilde{z}\right]\right) \tag{15}$$

Model with Symmetric and Complete Information: When all investors are informed about all asset-specific components and common components, our equations reduce to a form of the full-information Capital Asset Pricing Model (or full-info CAPM), expressed in term of prices, and adjusted for supply uncertainty. See Internet Appendix G for details of related calculations. The appendix also shows the full-info CAPM adjusted for supply uncertainty and expressed with covariance terms—a form that is more familiar to financial economists.

$$E\left[\tilde{P}^0\right] = \frac{1}{R}\left(E\left[\tilde{P}^1\right] - a\boldsymbol{\Sigma}_\varepsilon E\left[\tilde{z}\right]\right) \tag{16}$$

Information Discount Factor: We define the "information discount factor" (or DF_{info}) as the difference between the price discounts shown in Equations (15) and (16). The DF_{info} represents the amount an asset's price at date 0 is below its expected future value due to agents not having full information about future payoffs.

$$DF_{info} \equiv \frac{a}{R}\mathbf{V}_N E\left[\tilde{z}\right] - \frac{a}{R}\boldsymbol{\Sigma}_\varepsilon E\left[\tilde{z}\right]$$

$$= \frac{a}{R}(\mathbf{V}_N - \boldsymbol{\Sigma}_\varepsilon)E\left[\tilde{z}\right] \tag{17}$$

In a single-asset model with no factor structure, the information discount factor is proportional to the difference between the market's average uncertainty about future payoffs ($\mathbf{V_N}$) and residual uncertainty about the same payoffs ($\boldsymbol{\Sigma}_\varepsilon$). This difference is a signal-to-noise measure. When the difference is small, investors have a lot of information about future payoffs, the DF_{info} is low, and prices are high. Note that $DF_{info} \geq 0$ as the market is always bounded in its assessment of future payoffs by $\boldsymbol{\Sigma}_\varepsilon$.

In a multi-asset model with uncorrelated residual uncertainties and no factor structure, the single-asset intuition discussed in the paragraph above continues to hold. The diagonal matrix ($\mathbf{V}_N - \boldsymbol{\Sigma}_\varepsilon$) represents a series of signal-to-noise differences.

In a multi-asset model with correlated residual uncertainties and/or a factor structure, the information discount factor can be driven by both the asset-specific components of payoffs and common factors. The matrix ($\mathbf{V}_N - \boldsymbol{\Sigma}_\varepsilon$) can still be roughly interpreted as signal-to-noise differences. However, the matrix is no longer diagonal, which means that covariance terms affect the DF_{info}. Section 3. of this paper numerically analyzes asset prices in an effort to better understand the role of the covariance terms.

Risk premia are present in Albuquerque, Bauer, and Schneider (2009). Equation (18) on page 24 of the ABS paper gives an expression for an average stock price (specifically stock "j"). ABS mention that "the mean price is thus the mean dividend less a risk premium that is inversely related to ... the parameter that captures the accumulation of average public and private knowledge." The ABS equation is written in a way that is similar to

our Equation (15). The k_t coefficient in their paper represents the average precision of information in their model. As discussed above, we have an inverse matrix $(\mathbf{V}_N^{-1})$ which represents the precision of information for the "average" investor in our market. Both k_t and $\mathbf{V}_N^{-1}$ embed information precision about local and common components. However, the precise forms of these terms are obviously different since the information structure of ABS and of our paper are not the same.

We also note that there is a subtle difference between the ABS paper and ours. Both papers solve for risk premia (which can also be refered to as "discount factors"). The risk premia in both papers are functions of information-related variables as well as other variables such as noise and covariance. We differentiate our paper by solving for the difference between two risk premia.[13] If the two risk premia come from worlds that differ only in their information structures, then we are able to isolate price discounts that arrise solely due to information and not due to variables such as noise and covariance structures.

4.4 ANALYSIS OF INVESTOR HOLDINGS (PORTFOLIO CHOICE)

We analytically analyze relations between information structures and investor group n's holdings of risky assets. We vary structures, leave the model parameters constant, and measure ex-ante holdings. To do this, we take expectations of Equations (5) and (6) and rearrange terms to give:

$$
\begin{aligned}
E\left[\tilde{X}_n\right] &= a^{-1}\mathbf{V}_n^{-1}\left(E\left[\tilde{P}^1\right] - RE\left[\tilde{P}^0\right]\right) \\
&= \mathbf{V}_n^{-1}\mathbf{V}_N E\left[\tilde{z}\right]
\end{aligned}
\tag{18}
$$

In a single-stock world with no common components, investor n's holdings depend on the ratio of the market's uncertainty about the future payoff $(\mathbf{V}_N)$ to his own uncertainty about the same payoff $(\mathbf{V}_n)$. The higher the investor's uncertainty relative to the market, the lower the ratio, and the lower the weight of the asset in his portfolio.

In a multi-asset framework with uncorrelated residual uncertainty and no common components, the matrices $(\mathbf{V}_N)$ and $(\mathbf{V}_n)$ are diagonal. The term $\mathbf{V}_n^{-1}\mathbf{V}_N$ represents a series of uncertainty ratios. The same intuition described in the paragraph above holds.

In a multi-asset model with correlated residual uncertainties and/or a factor structure of payoffs, thinking about $\mathbf{V}_n^{-1}\mathbf{V}_N$ as a ratio of two uncertainty measures provides rough intuition only. However, the ratio of two matrices includes covariance terms relating to uncertainty about assets' payoffs. Investor n's holdings of a specific asset now depends on his uncertainty about the asset's payoffs, his uncertainty about other assets' payoffs, and other investors' uncertainty about all assets (including the asset in question). These uncertainties can arise from information asymmetries about the asset-specific components of payoffs and/or common components.

4.5 CHECKS AND RESTRICTIVENESS OF ASSUMPTIONS

Table 2 shows there are 14 structures that allow for closed-form solutions. For each of these 14 structures, and a set of parameters, we verify that the term $(\mathbf{\Psi}^{-1} + \mathbf{C}'\mathbf{\Sigma}_\varepsilon^{-1}\mathbf{C})$ is

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[13] In our paper, these two premia come from: (1) a world with symmetric and complete information and (2) a world with disperse information.

in fact a g-matrix. Using a set of parameters, we confirm that numerical and closed-form solutions give the same asset prices and holdings. Internet appendix J has additional notes on the closed-form solutions.

We evaluate the restrictiveness of our assumptions by inspecting results presented in Tables 2, 3, and 4. Looking at the results in Table 3, we see the 14 structures that allow for closed-form solutions generate price discounts ranging from 0.00 to 137.51. That is, the structures for which we can generate closed-form solutions offer a full range of price discounts. The structures for which we <u>cannot</u> generate closed-form solutions are not those with the largest price discounts. We conclude that we are not missing extreme results.

Looking at Table 4, we see the structures that allow for closed-form solutions generate holdings dispersion measures ranging from 0.000 to 0.168. The structures for which we cannot generate closed-form solutions are not those with the largest holdings dispersion measures. We do not appear to be limited in our ability to analyze (in closed-forms) structures that produce extreme results.

5. Holdings Dispersion and Home Bias

We consider a financial economist who wishes to better understand holdings dispersion across global equity portfolios. This paper's introduction mentions foreign ownership statistics related to publicly-traded German firms. From the perspective of the average German stock, foreign funds own 3.66% of the shares outstanding. For one quarter of German stocks, the same funds hold less than 0.01% of the equity in aggregate. For the upper quarter of German stocks, foreign funds hold at least 4.35% of the equity. If all foreign funds were market capitalization-based indexers, we should see them holding same percentage ownership of each German company.

Our model is able to generate large dispersions of foreign ownership in a way other models are not able to. A numerical example helps to make this point. We consider a four-country example (Germany, Japan, the UK, and the USA). Each country has a single stock. Investors have asset-specific information about their own country's stock. There are two common components. UK investors have information about the first component and US investors have information about the second. We measure home bias in the typical manner (see the equations directly below).[14] In Equation (20), we focus on US investors' holdings of German shares.

$$HB^j = 1 - \left[\frac{\sum_{i \neq j} \textit{Weight of country i's shares in investor-group j's portfolio}}{\sum_{i \neq j} \textit{Weight of country i's shares in the world market portfolio}} \right] \quad (19)$$

$$HB^{US\ Investors}_{German Shares} = 1 - \left[\frac{\textit{Weight of German shares in US investors' portfolios}}{\textit{Weight of German shares in the world market portfolio}} \right] (20)$$

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[14] Home bias is typically discussed in reference to an over-weighting of own-country assets. However, the financial economics literature also evaluates home bias as an under-weighting of foreign country assets. The measure used in this example follows convention. We thank Bruno Solnik for suggesting we use the first equation above to quantify home bias.

The first equation shows a general home bias expression for investors from country j. The second expression shows a type of *partial* home bias. In this example, we choose to focus on the US investors' holdings of the German stock and note there are eleven other possible measures of home bias (UK investors' holdings of Japanese stocks, etc.) The measure from the second equation is positive when US investors place a lower weight on the German stock than exists in the world market portfolio. When the measure is negative, we say there is reverse home bias and US investors place a higher weight on the German stock (within their portfolios) than does the world market portfolio. We graph results in Figure 2.

[Insert Figure 2 About Here]

Figure 2 is key to understanding our results and provides a summary of our contributions. In the figure, the X-axis shows the amount of asset-specific information in the German stock.[15] The four isobars (graph lines) represent increasing exposure of the German stock to the second common component (for which the US investor has valuable information).

The figure has several points of interest. First, points on the x-axis represent a portfolio weight equal to the market capitalization weight. Most points in the figure do not plot on the x-axis, thus we conclude that some amount of home bias is prevalent in our example. Second, some points plot far from the x-axis indicating portfolio weights that can be significantly different from market capitalization weights.

Third, we see that low levels of asset-specific information and high factor loadings can lead to reverse home bias (the US investor overweights the German stock relative to the world market portfolio). Reverse home bias is present whenever one of the graph lines falls below the X-axis. Fourth, on the figure's far left, we see that when there is zero loading on the second factor and no asset-specific information, the US investor holds the German stock at its market capitalization weight (the red line touches the origin).

Fifth, for a given level of asset-specific information (say "3") we see a range of home bias (measured as the difference between the red and blue lines.) We have added a dashed vertical line to signify when asset-specific information is 3 on our zero to 10 scale. Economically, the blue dot represents a world in which the US investor has 10% more weight in German shares than the world market portfolio would predict. The red dot represents a world in which the US investor has 40% less weight in German shares than the world market portfolio would predict.

Figure 2 highlights our concept of factor-based holdings dispersion. For a given level of asset specific-information (say 3) we can consider different amounts of cross-border holdings by looking up and down the dashed vertical line. US investors hold fewer German shares (home bias is larger) when the German stock does not load on the common component for which US investors' have valuable information. US investors exhibit reverse home bias when the German stock loads heavily on the common component. Given that there are many listed stocks in Germany, we expect US investors to exhibit *different* amounts

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[15] For this figure we consider values of θ ranging from 0.0 to 10. All four stocks have a 0.1 loading on the first factor which is known by the UK investors. Three stocks have a 2.0 loading on the second factor, while the German stock has loadings of 0, 1, 2, and 4 on the second factor. Since the second factor is known by US investors, this information structure gives US investors an increasing informational advantage as the second factor loading increases.

of home bias across stocks—hence their holdings can be described as disperse relative to CAPM weights. Our paper provides insights into how one might quantify the amount of dispersion and allow straightforward comparisons across countries. The distance between the two dots in Figure 2 helps quantify the economic significance of factor-based dispersion in our model.

Figure 2 also highlights differences between our paper and existing work. The model in Albuquerque, Bauer, and Schneider (2009), for example, considers only a single factor loading of one. Look at our figure and consider only the beta $= 1$ line. Relative to the CAPM, the beta $= 1$ line shows home bias measures ranging from -0.27 to $+0.50$. What cannot be shown when studying only the beta $= 1$ line, is that different home bias levels can exist for stocks with the same amount of asset-specific infomation (seen by going up and down the dashed vertical line.) The discussion we provide about the empirically observed dispersion of German stock holdings is thus difficult to justify without making assumptions about the level of asset-specific information for each and every stock.

Finally, future empirical studies could estimate factor loadings and make an assumption about which investors might have information about different common components. One could then test whether, cross border holdings of investors from country "j" do, in fact, vary with the absolute levels of stock i's factor loadings.

6. Conclusion

This paper proposes a multi-asset, rational expectations equilibrium model in which agents are asymmetrically informed about asset-specific and common components of payoffs. Our model allows agents to have asset-specific information and/or common component information. The model produces closed-form solutions for asset prices as well as for the holdings of individual agents.

Our solution for equilibrium prices is general and can be applied to numerous information structures. We solve the model for the case in which all investors have symmetric and complete information. We solve for other cases wherein investors are asymmetrically informed and/or do not have complete information. Our analysis leads to a closed-form solution for the information discount factor (or DF_{info}), which is the amount by which equilibrium prices are reduced due to agents not having full information about assets' future payoffs. The DF_{info} can be used to quantify the degree of informational frictions in the economy. A higher degree of informational frictions leads to a higher DF_{info} and lower prices.

The first paragraph of this paper asks: How do market prices differ when one group of investors has all common-component information compared with situations when different groups have information about different common components? We now have an answer— aggregate prices in the first structure are 8.06% higher than in the second structure.[16] The general form of our solutions allows us to ask and answer a host of additional questions. For example, how do prices change if the two groups without common-component information

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[16] From Tables 2 and 3, the structure called "ABS-Inspired v1" endows Group D with all common-component information and has a market capitalization of \$257.35. The structure "ABS-Inspired v3" endows Groups A, B, and C with information about f_1 and Group D with information about f_2 and has a market capitalization of \$238.16. The first value is 8.06% higher than the second.

share their information?[17] We can even ask: For a given structure, how do prices vary as factor loadings change? Expanding Table 4 to include market capitalizations (associated with the three different levels of factor loadings) would quickly show answers to such a question. Finally, and for a given set of parameters, the ability to model different structures allows us to say something about the impact of asset-specific information vs. common-component information.

In the introduction, we ask why some 401(k) plans invest heavily in their own company's stock. Consider information structures such as those shown in Figure 1, where employees are partitioned into four groups, and there are four companies and two common components. If investors feel they have superior asset-specific information, they may want to increase the weight of their own company's stock. However, investors will also consider how important economy-wide factors are to the stock. If factor loadings are large in absolute magnitudes and if other investors are likely to have valuable information about the factors, employees will decrease the weight of the stock.

We can also discuss the following question: Why do international mutual funds invest heavily in some foreign equities but not in others? In a simple framework such as in Gehrig (1993), investors are generally assumed to have information about their home country's assets (or to view the assets as less risky). The last figure in Internet Appendix H depicts such an information structure. Such a structure generates home bias indicating that an investor holds more of an asset than he would if he invested in the world market portfolio. However, generating large levels of holdings dispersion (for foreign stocks) is difficult in these structures. Generally, investors overweight their own country's stocks and (equally) underweight the stocks from all other countries. Our model allows us to understand dispersion across the (under-weighted) stocks from other countries.

In addition to home bias, our model can generate reverse (or negative) home bias indicating an investor underweights his home country's assets relative to world market portfolio weights—see Bravo-Ortega (2005). A strength of our model is its ability to produce large variations in home bias as well as its ability to produce reverse home bias. The advantage stems from thinking about economy-wide information. If common components play a large role in determining an asset's payoff, those with information about the components are likely to overweight the asset regardless of whether it comes from one's home country or from a foreign country.

Studies related to home bias can/do focus on areas beyond international portfolio choice. Examples include individual ownership of own-company stock (such as 401(k) plans), ownership patterns determined by investors' job locations/industries, ownership patterns determined by stocks' industries, and intra-national home bias as in Coval and Moskowitz (1999). All of these examples have an inherent tension between the value of company-specific information, the value of common-component information, and the desire to diversify. Our model provides insights into all situations. For example, and given the right data, one could test whether stocks with high levels of ownership (by investors working in the same industry) load more or less significantly on industry-wide factors.

There are a number of additional avenues for potential future research. First, one could try to extend our model to multiple periods. This would provide expressions for net trad-

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[17] The structures "ABS-Inspired v1" and "ABS-Inspired v2" help answer this question. In the "v2" structure, Groups A, B, and C do *not* share their asset-specific information. In the "v1" structure, the groups do share the asset-specific information. When sharing, the market capitalization is 0.92% higher than when not sharing ($257.35 vs. $255.00).

ing as in Brennan and Cao (1997), and would suggest empirical tests based on trading (as opposed to holdings) data. Second, one could work to devise methods of empirically identifying different information structures. While it would be no small task, structures could then be used to test relative asset prices using expressions in this paper. Third, our model may be adapted to develop a better understanding of partially segmented markets. In such cases, information reflects the "friction" that segments markets. One may be able to model groups of investors who face low frictions only when trading securities from their home country, groups of investors who face low frictions when trading securities in a contiguous block of countries (a geographic region), or groups of investors who face low frictions when investing in any global security. None of the three extensions is likely to be easy—but all are potentially interesting.

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Table 1: Variable Descriptions and Definitions

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a	The coefficient of constant absolute risk aversion.
$\tilde{f}$	Vector of K common components with $\tilde{f} = (\tilde{f}_1 \ldots \tilde{f}_K)'$.
r_f	Riskfree rate with $R = (1 + r_f)$.
$\tilde{w}_i^t$	Wealth of investor i and date t.
$\tilde{z}$	Vector of per capital asset supplies with $\tilde{z} = (\tilde{z}_1 \ldots \tilde{z}_J)'$.

$\tilde{\epsilon}$	Vector of residual uncertainties with $\tilde{\epsilon} = (\tilde{\epsilon}_1 \ldots \tilde{\epsilon}_J)'$.
$\tilde{\eta}$	The "stacked" vector of asset-specific and common component with $\tilde{\eta} = (\tilde{\theta}' \tilde{f}')'$.
λ_n	The fraction of total investors (I) in group n such that $\sum_{n=1}^{N} \lambda_n = 1$.
$\tilde{\theta}$	Vector of assets specific component of payoffs with $\tilde{\theta} = (\tilde{\theta}_1 \ldots \tilde{\theta}_J)'$.

$\mathbf{\Psi}$	The variance-covariance matrix of $\tilde{\eta}$ conditional on observing the equilibrium price vector at date 0.
$\mathbf{\Psi}_n$	Matrix equal to: $\mathbf{M}_n \mathbf{\Psi} \mathbf{M}_n'$
$\mathbf{\Sigma}_\epsilon$	The covariance matrix of the residual uncertainty.
$\mathbf{\Sigma}_\theta$	The covariance matrix of the asset-specific component of payoffs.
$\mathbf{\Sigma}_z$	The covariance matrix of the supply shocks.

Table 1: Continued

A_0, $\mathbf{A}_1$, $\mathbf{A}_2$	One vector and two matrices of constants in the price equation.
B_{0n}, $\mathbf{B}_{1n}$, $\mathbf{B}_{2n}$	One vector and two matrices of constants in the price equation.
$\mathbf{B}$	$J \times K$ matrix of factor loadings.
$\mathbf{C}$	$J \times (J + K)$ block-diagonal matrix consisting of $\mathbf{I}_J$ and $\mathbf{B}$.
$\mathbf{D}$	Defined as $\mathbf{D} \equiv \sum_{n=1}^{N} \lambda_n \mathbf{D}_n$.
$\mathbf{D}_n$	$\mathbf{D}_n$ is a diagonal matrix of order $J + K$ with ones on the main diagonal if investors in group n have asset-specific or common-component information.
g-matrix	A square matrix ($\mathbf{G}$) of order $J + K$ which satisfies $g(\mathbf{G}) = \mathbf{G}$.
I	Total number of investors.
$\mathbf{I}_J$	A $J \times J$ identity matrix.
$\mathbf{I}_K$	A $K \times K$ identity matrix.
J	Number of assets.
J_n	Number of assets for which investors in group n have asset-specific information.
K	Number of common components (factors) in the economy.
K_n	Number of common components for which investors in group n have information.
$\mathbf{M}$	Matrix equal to: $\mathbf{U}\mathbf{Q}\mathbf{U}' + \mathbf{\Sigma}_z$
$\mathbf{M}_n$	This matrix is obtained by eliminating the null (zero) rows of $\mathbf{D}_n$.
N	Number of non-overlapping groups.
$\tilde{P}^0$	Vector of equilibrium prices at date 0 with $\tilde{P}^0 = (\tilde{P}_1^0 \ldots \tilde{P}_J^0)'$.
$\tilde{P}^1$	Vector of final payoffs with $\tilde{P}^1 = (\tilde{P}_1^1 \ldots \tilde{P}_J^1)'$.
$\mathbf{Q}$	The variance-covariance matrix of $\tilde{\eta}$.
R	Gross riskfree rate with $R = (1 + r_f)$.
$\mathbf{U}$	Matrix defined as: $\mathbf{U} \equiv \mathbf{A}_2^{-1}\mathbf{A}_1$.
$\mathbf{V}_N$	Conditional variance of $\tilde{P}^1$ from "average" investor's point of view.
$\mathbf{V}_n$	Conditional variance of $\tilde{P}^1$ from investor n's point of view.
$\tilde{X}_n$	The holdings of investor group n.

Table 2: Summary of Different Information Structures

This table overviews the different information structures studied in our numerical analysis. We consider four groups of investors labeled A, B, C, and D, four assets numbered 1, 2, 3, and 4, and two factors f_1 and f_2. For each group of investors and each structure, we use asset numbers to note whether the investor group has asset-specific information. We use the letter f plus the factor numbers to note whether the investor group has factor information. We model information structures consistent with those studied in the frictionless/full-information CAPM, symmetric structures, Albuquerque, Bauer, and Schneider (2009) inspired structures, Kodres and Pritsker (2002) inspired structures, as well as some additional information structures.

	Investor Group A	Investor Group B	Investor Group C	Investor Group D	Closed-Form Solution Available
	$1,2,3,4,f_1,f_2$	$1,2,3,4,f_1,f_2$	$1,2,3,4,f_1,f_2$	$1,2,3,4,f_1,f_2$	
Full-Info CAPM	$1,2,3,4,f_1,f_2$	$1,2,3,4,f_1,f_2$	$1,2,3,4,f_1,f_2$	$1,2,3,4,f_1,f_2$	Yes
Symmetric v1	$1,2,3,4,f_1$	$1,2,3,4,f_1$	$1,2,3,4,f_1$	$1,2,3,4,f_1$	Yes
Symmetric v2	$1,2,3,4,f_2$	$1,2,3,4,f_2$	$1,2,3,4,f_2$	$1,2,3,4,f_2$	Yes
ABS-Inspired (base)	$1,3,4$	$1,3,4,f_1,f_2$	$2,3,4$	$2,3,4,f_1,f_2$	No
ABS-Inspired v1	$1,2,3$	$1,2,3$	$1,2,3$	$4,f_1,f_2$	Yes
ABS-Inspired v2	1	2	3	$4,f_1,f_2$	Yes
ABS-Inspired v3	$1,2,3,f_1$	$1,2,3,f_1$	$1,2,3,f_1$	$4,f_2$	No
ABS-Inspired v4	1	2	3	$4,f_1$	Yes
ABS-Inspired v5	1	2	3	$4,f_2$	Yes
Kodres & Pritsker (base)	$1,2,3,4$	$1,2,3,4$	$1,2,3,4$	$1,2,3,4$	Yes
Kodres & Pritsker v1	$1,2$	$2,3$	$3,4$	$4,1$	No
Kodres & Pritsker v2	1	2	3	4	Yes
Additional v1	1	$2,3,4,f_1,f_2$	$2,3,4,f_1,f_2$	$2,3,4,f_1,f_2$	Yes
Additional v2	1	2	$3,4,f_1,f_2$	$3,4,f_1,f_2$	Yes
Additional v3	1	$1,2,f_1$	$1,2,3,f_2$	$1,2,3,4,f_1,f_2$	No
Additional v4	1	2	$3,f_1,f_2$	$4,f_1,f_2$	No
Additional v5	$1,f_1,f_2$	$1,2,f_1$	$1,2,3,f_2$	$1,2,3,4$	No
Additional v6	$1,2$	3	$4,f_1,f_2$	$4,f_1,f_2$	Yes
Additional v7	$1,2,3$	$1,2,3$	$4,f_1,f_2$	$4,f_1,f_2$	Yes
Additional v8	1	$2,3$	$2,3$	$4,f_1,f_2$	Yes

Table 3: Aggregate Market Capitalizations

This table summarizes the aggregate market capitalizations ("MC") of different information structures. The "Price Discount from CAPM" represents the difference between prices in a given structure and prices in the frictionless CAPM world. The quantity "Degree of Frictions" is defined as the ratio of the price discount to the market capitalization of the CAPM world.

	MC ($)	Price Discount from CAPM	Degree of Frictions
Full-Info CAPM	304.00	0.00	0.00
Symmetric v1	240.00	64.00	0.21
Symmetric v2	240.00	64.00	0.21
ABS-Inspired (base)	291.31	12.69	0.04
ABS-Inspired v1	257.35	46.65	0.15
ABS-Inspired v2	255.00	49.00	0.16
ABS-Inspired v3	238.16	65.84	0.22
ABS-Inspired v4	190.12	113.88	0.37
ABS-Inspired v5	190.12	113.88	0.37
Kodres & Pritsker (base)	176.00	128.00	0.42
Kodres & Pritsker v1	170.98	133.02	0.44
Kodres & Pritsker v2	166.49	137.51	0.45
Additional v1	297.55	6.45	0.02
Additional v2	285.04	18.96	0.06
Additional v3	281.27	22.73	0.07
Additional v4	279.60	24.40	0.08
Additional v5	266.64	37.36	0.12
Additional v6	278.53	25.47	0.08
Additional v7	279.39	24.61	0.08
Additional v8	255.94	48.06	0.16

Table 4: Holdings Dispersion

This table summarizes the dispersions across investors' holdings that are associated with different information structures. The three columns of numbers show our measure of holdings dispersion when all the elements in the **B** matrix are 0.5, 1.0, or 1.5. We compare portfolio weights to the overall weights in the market portfolio. Holdings dispersion is defined as the root mean squared error (RMSE).

	B(0.5)	**B**(1.0)	**B**(1.5)
Full-Info CAPM	0.000	0.000	0.000
Symmetric v1	0.000	0.000	0.000
Symmetric v2	0.000	0.000	0.000
ABS-Inspired (base)	0.051	0.059	0.065
ABS-Inspired v1	0.060	0.094	0.144
ABS-Inspired v2	0.088	0.120	0.168
ABS-Inspired v3	0.058	0.060	0.061
ABS-Inspired v4	0.085	0.088	0.090
ABS-Inspired v5	0.085	0.088	0.090
Kodres & Pritsker (base)	0.000	0.000	0.000
Kodres & Pritsker v1	0.077	0.079	0.079
Kodres & Pritsker v2	0.085	0.086	0.086
Additional v1	0.063	0.068	0.070
Additional v2	0.088	0.110	0.127
Additional v3	0.033	0.026	0.021
Additional v4	0.086	0.102	0.116
Additional v5	0.058	0.091	0.135
Additional v6	0.079	0.099	0.117
Additional v7	0.065	0.087	0.106
Additional v8	0.082	0.114	0.162

Figure 1
Two Possible Information Structures

These diagrams depict worlds with four groups of investors labeled A, B, C, and D. There are four assets, depicted by factories, and two factors denoted "f_1" and "f_2". Solid lines from factors to assets indicate that payouts are determined by an underlying factor structure. Dotted/dashed lines from investor groups to either assets or to factors indicate investor groups possess information on these assets or factors.

Panel A: Symmetric and Complete Information (Frictionless or Full-Info CAPM)

The diagram depicts the information structure in a full-info CAPM world. In this diagram, each group of investors has information about each asset and each factor. No investor has an information advantage or disadvantage. All possible information about each asset's payoff (other than the residual uncertainty) is known.
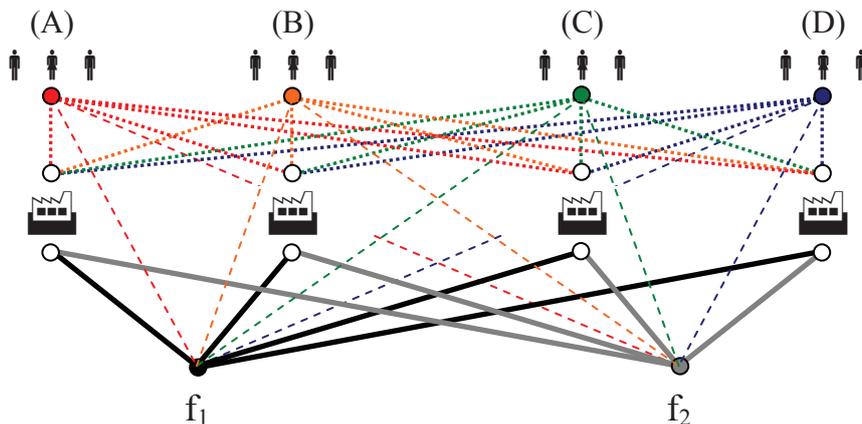
Panel B: Additional Structure v6

The diagram depicts the information structure that is denoted "Additional v6" in the tables. Investors group A has asset-specific information about assets 1 and 2 but they do not have any factor information. Investor group B has asset-specific information about asset 3. Investor groups C and D have asset-specific information about asset 4 and information about both factor 1 and factor 2. Both factors f_1 and f_2 affect asset returns.
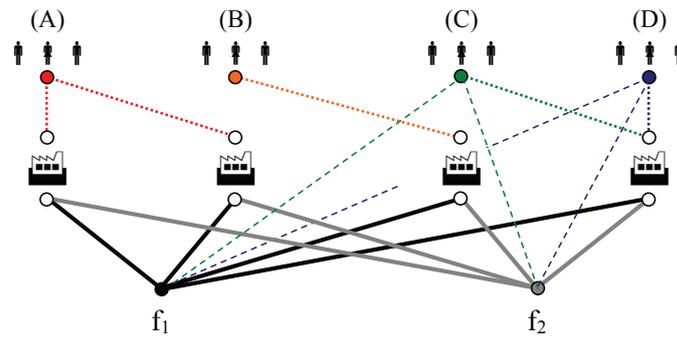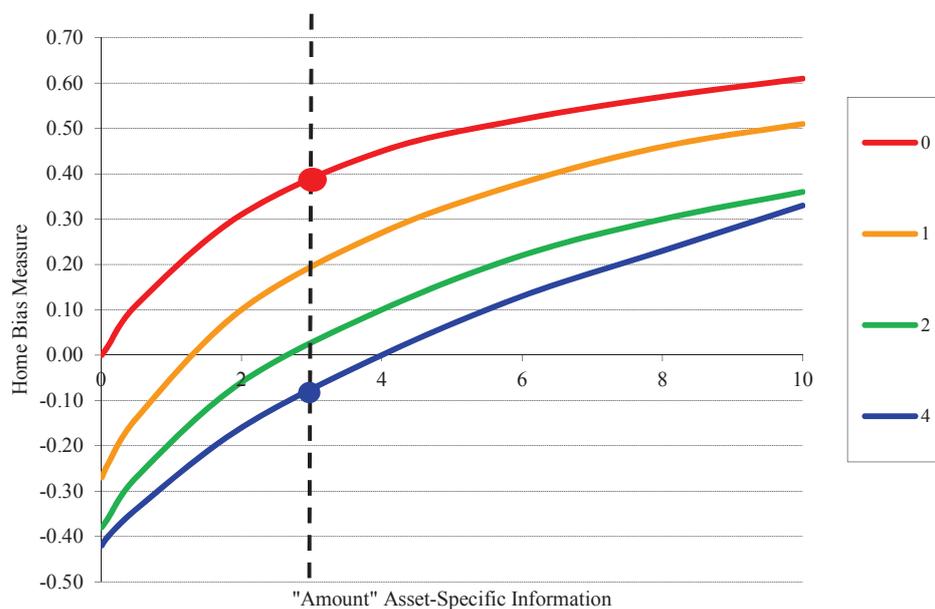
Figure 2
Home-Bias (Holdings) Dispersion

The figure depicts different levels of home-bias for a sample of simulated data. We consider a four-country example (Germany, Japan, UK, USA). Each country has a single stock. While home bias is typically shown as an over-weighting of own-country assets, it can also be evaluated as an under-weighting of foreign country assets. In this example, we look at US investors' holdings of the German stock. A positive value represents home bias, while a negative value represents reverse home bias. The X-axis shows the amount of asset-specific information in the German stock. The four isobars (lines) represent increasing exposure of the German stock to a common component—i.e., factor loadings are $\{0, 1, 2, 4\}$. In this example, the US investor has valuable information about the common component. Points on the x-axis represent a portfolio weight equal to the CAPM weight.